ECN594: Math Bootcamp

Practice Questions I

August 2022

1 Logic

1. Find the contrapositive of the statement

$$A \wedge B \Longrightarrow C,$$

and find two sufficient conditions for this statement.

2. Let P, Q, R be statements. Assume the following statement is true: *"if P is true and Q is not true then R is true."*

a) Rewrite the statement above in logical operators.

b) What can we infer about statements P and Q if R is not true?

3. Let A and B be nonempty sets. Negate the statement

$$(\exists x \in A, \forall y \in B)\ [P(x, y)].$$

4. Let n be a positive integer, show that $n < 2^n$.

5. Prove the following result:

The well-ordering principle. *Every nonempty finite set of positive integers contains a least element.*

2 Set Theory

1. Suppose $A \subseteq B \subsetneq C$, prove that $A \subsetneq C$ by contraposition.

2. Prove part (2) of Proposition 2.

3. Show that a necessary and sufficient condition for that

$$(A \cap B) \cup C = A \cap (B \cup C)$$

is that $C \subseteq A$.

4. Prove Claim 2.

5. Identify which of the following subsets of $\mathbb{R}^2$ is a function or not. Explain.

a) $\{(1,2),(2,3),(2,4),(3,3)\}$.

b) $\{(1,2),(2,3),(3,3),(4,1)\}$.

c) $\{(x,y) \in \mathbb{R}^2 : y^2 = 1 - x^2\}$.

d) $\{(x,y) \in [0,1]^2 : x = y, x \in [0,1)\} \cup \{(1,0)\}$.

For each of the functions, determine whether it is injective, surjective, and/or bijective.

6. Let $f : X \longrightarrow Y$ be a function, where X and Y are nonempty sets. Prove the following two assertions:

a) A necessary and sufficient condition that

$$f(X \backslash A) \subseteq Y \backslash f(A) \quad \text{for all } A \subseteq X$$

is that f is injective.

b) A necessary and sufficient condition that

$$Y \backslash f(A) \subseteq f(X \backslash A) \quad \text{for all } A \subseteq X$$

is that f is surjective.

Therefore, $f(X \backslash A) = Y \backslash f(A)$ if and only if f is bijective.

7. Let X, Y, Z be nonempty sets, and let $f : X \longrightarrow Y$, $g : X \longrightarrow Y$, $u : Y \longrightarrow Z$, $v : Y \longrightarrow Z$ be functions. Show that

i) if f and u are both surjective, then so is $u \circ f$;

ii) if f and u are both injective, then so is $u \circ f$;

iii) if f is surjective and $u \circ f = v \circ f$, then $u = v$; and

iv) If u is injective and $u \circ f = u \circ g$, then $f = g$.