Externalities, Public Goods, and Lindahl Equilibrium*

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1 Introduction

At the beginning of this semester, we saw a tight connection between Walrasian equilbria and Pareto optimality: for example, the first welfare theorem tells us that, under some (mild) assumptions on preferences, every Walrasian equilibrium is Pareto optimal. One might be, hence, tempted to interpret the first welfare theorem as an endorsement of market economies. It is important to note that, however, the theorem keeps silent on equity, and general equilibrium models do not seriously address many driven forces of economic performance. Even if we put these concerns aside, in some situations, some of the (possibly implicit) assumptions of the first welfare theorem may fail to hold; as a consequence, market equilibria cannot be relied on to yield Pareto optimal outcomes. If this is the case, we say that market *fails*.

In particular, one important hidden assumption is that there are no externalities in consumption or production. Recall that, in an Arrow-Debreu economy, the preferences of a consumer is defined solely over the set of conceivable consumption bundles that she herself may decide to consume; similarly, the production of a firm depend only on its own input choices. However, in reality, a consumer's well-being might be affected by the actions of the others in the economy: it might matters to her that what her neighbors consumer does, what the firm down the street does, and so on. For example, you did not sleep well because your roommate's consumption of loud music at 4AM, and thus you might not be able to learn much from your afternoon microeconomic theory class. To incorporate these into our models, we need to define an agent's (could be a consumer or a firm) preferences or production set over both her actions and those creating an external effect on her. We show in a very simple

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partial equilibrium model that Walrasian equilibria are not necessarily Pareto optimal when externalities are present. As you could expect, we are going to consider some solutions to the externality problem proposed by some economists.

Public goods, as the name suggests, are commodities that have an inherently "public" character, in the sense that consumption of a unit of the good by one agent does not preclude its consumption by another. Examples around: national defense, highways, knowledge, projects that improves air quality, etc. The private provision of public goods generates a special type of externality: if one individual provides a unit of public good, all individuals benefit from it. As a result, private provision of public goods is typically inefficient.

However, by modifying the model, we are able to use another equilibrium concept, known as Lindahl equilibrium, to restore optimality of market outcome. In a Lindahl equilibrium, if one agent engages in an externality-generating activity, a transfer has to be made to all other agents who are possibly affected.

It is worth noting that externality is only a specific source of *market failure*. Other sources of market failure include *market power*, which you have seen last semester, and *information asymmetry*, which is the main topic of the second half of this class.

2 Externatilies

Intuitively, an externality is simply a case in which the economic activities of one party—a consumer or a firm—have a direct impact on the utilities or production possibility sets of others, where "direct" means that we exclude all effects that are mediated by prices. In this section, we show that, in a very simple partial equilibrium setting, when externalities are present, Walrasian equilibria may not be optimal. We also discuss some possible solutions to the externalities problem.

For simplicity, let us go back to a partial equilibrium setting for a moment. We only consider two consumers,[1] and each of them only cares about two things: an action h taken by Consumer 1, and a numeraire (say money); their preferences are quasilinear in the numeraire.[2] Then for $i = 1, 2$, Consumer i's utility function can be written as

$$v_i(h, w_i) = \varphi_i(h) + w_i.$$

We further assume that, for $i = 1, 2$, $\varphi_i(\cdot)$ is twice continuously differentiable and strictly

[1] Interested readers are referred to Section 11.D of Mas-Collel et al. (1995) for a more general treatment for externalities in partial equilibrium setting.

[2] See Section 11.B in Mas-Collel et al. (1995) for a "microfundation" for these preferences that involves L consumption goods.

concave; equivalently, $\varphi_i''(\cdot) < 0$. Observe that Consumer 1 's choice of h affects Consumer 2's well-being, hence it generates an externality. In the context of the example in Section 1, Consumer 1 and Consumer 2 are roommates, and h could be a measure of how loud the music that Consumer 1 plays at night is.

2.1 Suboptimality of Walrasian Equilibria

In a Walrasian equilibrium, both consumers maximize their perspective utilities; hence it must be that consumer 1 chooses $h \in \mathbb{R}_+$ to maximize $\varphi_1(h)$. Because φ_1 is strictly concave, the equilibrium level h^* can be derived from the first-order condition (FOC):

$$\varphi_1'(h^*) \leq 0, \quad \text{with equality if } h^* > 0.$$

However, the Pareto optimal level h^o must maximize the *joint surplus* of the two consumers (we ignore w_i's in the problem since they are not affected by h), hence it solves

$$\max_{h \in \mathbb{R}_+} \varphi_1(h) + \varphi_2(h),$$

and the FOC is

$$\varphi_1'(h^o) \leq -\varphi_2'(h^o), \text{ with equality if } h^o > 0. \tag{1}$$

Thus, when externalities are present, namely $\varphi_2'(h) \neq 0$ at all $h \in \mathbb{R}_+$, the equilibrium level of h is not optimal (that is, $h^* \neq h^o$) unless $h^* = h^o = 0$. If $h^* > 0$ and $h^o > 0$, so the solutions are interior, we have $\varphi_1'(h^*) = 0$, and $\varphi_1'(h^o) = -\varphi_2'(h^o)$. In particular,

- if $\varphi_2'(h) < 0$ for all $h \in \mathbb{R}_+$, we say that h generates a *negative externality*: we have $\varphi_1'(h^o) = -\varphi_2'(h^o) > 0$; then because $\varphi_1'(\cdot)$ is decreasing and $\varphi_1'(h^*) = 0$, this implies that $h^* > h^o$: the level of the activity that generates negative externalities is excessive in a Walrasian equilibrium;

- in contrast, when $\varphi_2'(h) > 0$ for all h, we say that h generates a *positive externality*; $\varphi_1'(h^o) = -\varphi_2'(h^o) < 0$ implies that $h^* < h^o$: the level of the activity that generates positive externalities is insufficient in a Walrasian equilibrium.

In a Walrasian equilibrium, the lone objective of a consumer who engages in an enternality-generating action is to maximize her own utility, so she does not care about whether her action affects the other consumer's well-being; nonetheless, Pareto optimality imposes restrictions on both consumers' well-being. Interestingly, Pareto optimality does not necessarily entail the complete elimination of externalities, even if a negative externality is present. In fact, the level of externality-generating activity is adjusted such that, the marginal benefit of an additional unit of the externality-generating activity to Consumer 1, $\varphi_1'(h)$, equals the marginal cost to Consumer 2, $-\varphi_2'(h)$.

2.2 Traditional Solutions to the Externality Problem

Having realized the inefficiency of the competitive market outcome in the presence of externalities, many economists have proposed their solutions to the problem. To make life easier, we assume, for the next few paragraphs, that h generates negative external effects, so $h^o \leq h^*$. The conceptually simplest solution is, letting the government simply mandates that h can be no larger than h^o. With this constraint, Consumer 1 would indeed fix the level of externality at h^o.

A second option, which also relies on the government, is to impose a tax on externality-generating activities, and thus restores optimality. This solution is known as *Pigouvian taxation*, after Pigou (1932). Let us go back to the simple model we discussed in Section 2.1, but suppose that, Consumer 1 has to pay tax t_P for each unit of h she consumes. Her problem becomes

$$\max_{h \in \mathbb{R}_+} \varphi_1(h) - t_P h,$$

and her optimal level, denote by h_P^*, is determined by the FOC

$$\varphi_1'\left(h_P^*\right) \leq t_P, \qquad \text{with equality if } h_P^* > 0.$$

Then by (1), to restore the optimal level h^o, it suffices to set $t_P = -\varphi_2\left(h^o\right)$.

Note that, the Pigouvian tax is exactly equal to the marginal externality evaluated at the optimal level h^o. In other words, it is exactly equal to the willingess to pay of reducing h slightly from h^o. With this tax, Consumer 1 is effectively led to carry out an individual cost-benefit computation that *internalizes* the externality she imposed on Consumer 2.

Another approach to the externality problem aims at a less intrusive form of intervention, seeking to ensure that conditions are met for the parties to themselves reach an optimal agreement on the level of the externality.

Suppose that we establish enforceable property rights with regard to the activity that generates an negative externality. Recall that, for Consumer 1, the marginal benefit from the externality-generating activity is

$$MB(h) = \varphi_1'(h);$$

and Consumer 2 suffers a marginal cost

$$MC(h) = -\varphi_2'(h).$$

Because we assumed that φ_i is strictly concave for $i = 1, 2$, $MB(h)$ is strictly decreasing, and $MC(h)$ is strictly increasing. We assume $MB(0) > MC(0)$ to avoid uninteresting cases; and to make our argument below more intuitive, we say that Consumer 1 and Consumer 2 are roommates, the former is a smoker, and the latter does not smoke. Then h is the amount of cigarettes that Consumer 1 smokes.

Say we assign the right to having a smoke-free environment to Consumer 2, that is, the non-smoker. In this case, Consumer 1 (the smoker) is unable to smoke without Consumer 2's permission. In exchange for a transfer to the non-smoker, the first cigarette should be smoked since $MB(0) > MC(0)$; and apparently, the non-smoker would be happy to accept any transfer $t_0 \in [MB(0), MC(0)]$. Hence, this deal making should continue as long as $MB(h) > MC(h)$, and stop at the level $\hat{h}$ where $MB(\hat{h}) = MC(\hat{h})$. Provided that the transfers do not impact either roommate's marginal cost or benefit, $MB(\hat{h}) = MC(\hat{h})$ implies $\varphi_1'(\hat{h}) = \varphi_2'(\hat{h})$, then by (1), we must have $\hat{h} = h^o$. Hence, the smoker would choose the efficient level of smoking and make some transfer payment to the nonsmoker. Note that an implicit assumption behind the above argument is that negotiations between the two consumers is costless.

Importantly, the precise allocation of these rights between the two consumers is inessential to the achievement of optimality. If we assign the right to smoke to Consumer 1, who is the smoker, efficiency would also be restored: the smoker would not consume the h^*-th cigarette because $MB\left(h^*\right) < MC\left(h^*\right)$, so she prefers to accept a transfer from the non-smoker instead; this process will continue until $h = h^o$, then $MB\left(h^*\right) = MC\left(h^*\right)$ and the efficient level is, again, achieved.

Our discussion above can be summarized by the following result, proposed by Coase (1960):

Coase Theorem. Assume that

(1) negotiation is costless,

(2) transfers do not affect any consumer's marginal values, and

(3) property rights are well-defined,

then the Pareto optimal outcome arises irrespective of who has property rights.

In the view of Coase (1960), in many cases, inefficiencies caused by externalities arise because the property rights are ill-defined.

2.3 Externalities and Missing Markets

Arrow (1969) suggests that, we may count the right to engage in the externality-generating activity as an extra product in the economy, for which there is currently no market. Note that, if markets for some goods are missing, the first welfare theorem may not hold. In the smoker and non-smoker example, we can assign one or the other agent with permits to smoke, and then create a market for trading these permits. Then the smoker is allowed to smoke h cigarettes now only if she owns h permits for producing smoke.

Formally, suppose that property rights are well defined and enforceable, and that a competitive market for the permits to engage in the externality-generating activity exists. Without loss of generality, we assume that Consumer 2 owns all the permits. Let p denote the price of the permit to engage in one unit of the externality-creating activity, say smoking. To choose how many units to purchase, Consumer 1 solves

$$\max_{h_1 \in \mathbb{R}_+} \varphi_1(h_1) - ph_1;$$

the FOC is

$$\varphi_1'(h_1^*) \le p, \qquad \text{with equality if } h_1^* > 0. \tag{2}$$

And to decide how many units to sell, Consumer 2 solves

$$\max_{h_2 \in \mathbb{R}_+} \varphi_2(h_2) + ph_2;$$

the FOC is

$$\varphi_2'(h_2^*) \le -p, \qquad \text{with equality if } h_2^* > 0. \tag{3}$$

In equilibrium, the market for permits must clear, so we must have $h_1^* = h_2^*$. Combining (2) and (3), we see that the level of rights traded in this competitive permits market, say $\tilde{h}$, must satisfy

$$p = \varphi_1'(\tilde{h}) = -\varphi_2'(\tilde{h}), \qquad \text{with equality if } \tilde{h} > 0,$$

where the second equality is exactly (1). Thus, we conclude that $\tilde{h} = h^o$, and the equilibrium price for the permit is $p = \varphi_1(h^o) = -\varphi_2(h^o)$. This result implies that, if a competitive market exists for externalities (more precisely, for the permits), optimality is restored.

3 Public Goods

In simple words, a public good is a commodity for which the use of a unit of the good by one agent does not preclude or decrease its use by other agents. Put differently, public goods are *nondepletable*: consumption by one agent does not affect the supply available to another. A good example is (at least most kinds of) knowledge: the fact that your colleagues have learned the second welfare theorem does not make that theorem unavailable to you. On the contrary, we call the usual goods, which have a depletable nature, *private goods*. Importantly, a public good need not to be desirable; that is, we may have public "bads" like pollution.

A distinction can also be made according to whether *exclusion* of an individual from the goods is possible. Of course, every private good is excludable, but public goods may or may not be. Being nondepletable, a patent is excludable; and a well-known example for nonexcludable public good is national defense.

Again, we consider a simple partial equilibrium model.[3] There are $I > 1$ consumers; similar to Section 2, we assume that, for each $i = 1, \ldots, I$, Consumer i only cares about the level of the public good and a numeraire, and her preferences are quasilinear in the numeraire.[4] Say we have q units of the public good, Consumer i's utility function is

$$V_i\left(q, m_i\right) = \phi_i(q) + m_i.$$

We assume that, for all $i = 1, \ldots, I$, $\phi_i''(\cdot)$ is twice continuously differentiable and strictly concave; thus, $\phi_i''(\cdot) < 0$.

Let the cost function of public good production be $c(\cdot) : \mathbb{R}_+ \to \mathbb{R}$, we assume that $c(\cdot)$ is twice continuously differentiable and strictly convex; that is, $c''(\cdot) > 0$. In what follows, we restrict our attention on a public good which is desirable and costly to produce. Hence, we further assume that $\phi_i'(\cdot) > 0$ for all i and $c'(\cdot) > 0$.[5]

3.1 Inefficiency of Private Provision of Public Goods

To find the Pareto optimal level of public good provision, it suffices to solve the aggregate surplus maximization problem:

$$\max_{q \in \mathbb{R}_+} \sum_{i=1}^{I} \phi_i(q) - c(q).$$

[3]For some discussion of the same problem in general equilibrium context, please see Example 16.G.3 in Mas-Collel et al. (1995).

[4]See Section 11.C in Mas-Collel et al. (1995) for a "microfundation" for these preferences that involves L consumption goods.

[5]The analysis is identical for the case that the public good is undesirable, and its reduction is costly: $\phi_i'(\cdot) < 0$ for all i and $c'(\cdot) < 0$.

Because $\phi_i(\cdot)$ is strictly concave for all $i = 1, \ldots, I$, and $c(\cdot)$ is strictly convex, the objective function is strictly concave. Hence, the necessary and sufficient FOC is

$$\sum_{i=1}^{I} \phi_i'(q^o) \leq c'(q^o), \qquad \text{with equality if } q^o > 0,$$

where q^o denotes the Pareto optimal quantity of the public good. At an interior solution, we have

$$\sum_{i=1}^{I} \phi_i'(q^o) = c'(q^o), \tag{4}$$

which says that at the Pareto optimal quantity of the public good, the sum of consumers' marginal benefits from the public good is set equal to its marginal cost. (4) is (the quasilinear and partial equilibrium case of) the *Samuelson condition*, the classic optimality condition for a public good, after Samuelson (1954).

In a Walrasian equilibrium, each consumer i takes the price p^*, and the amount of the public good purchased by other consumers as given, and choose the quantity that she purchase to maximize her utility. She solves

$$\max_{x_i \in \mathbb{R}_+} \phi_i\left(x_i + \sum_{k \neq i} x_k^*\right) - p^* x_i,$$

and her choice must satisfy the FOC

$$\phi_i'(x^*) \leq p^*, \qquad \text{with equality if } x_i^* > 0, \tag{5}$$

where $x^* = x_i^* + \sum_{k \neq i} x_k^*$ is the equilibrium level of the public good. And taking equilibrium price p^* as given, the firm which produces the public good must solve

$$\max_{q \in \mathbb{R}_+} p^* q - c(q);$$

hence firm's supply q^* must satisfy the FOC

$$p^* \leq c'(q^*), \qquad \text{with equality if } q^* > 0. \tag{6}$$

Lastly, market clearing requires $x^* = q^*$. Hence, if $q^* > 0$, it must be that $x_j^* > 0$ for some j, Then for some j, by (5) and (6), $\phi_j'(q^*) = c'(q^*)$; since $\phi_i'(\cdot) > 0$ for all $i = 1, \ldots, I$, we must have

$$\sum_{i=1}^{I} \phi_i'(q^*) > c'(q^*). \tag{7}$$

Comparing (4) and (7), since all $\phi_i(\cdot)$'s are strictly concave, $\sum_{i=1}^{I} \phi_i'(\cdot)$ is strictly decreasing, so $q^* < q^o$ whenever $q^o > 0$; that is, if the efficient level of (desirable) public good provision is strictly positive, the equilibrium level would be too low.[6]

Importantly, each consumer's purchase of the public good provides a direct benefit not only to the consumer herself but also to every other consumer, and hence purchasing the public good generates *positive externalities*. Therefore, as discussed in Section 2, the level of the activity that generates positive externalities would be insufficient in a Walrasian equilibrium. The failure of each consumer to consider the benefits for others of her public good provision is often referred to as the *free-rider problem*: each consumer has an incentive to enjoy the benefits of the public good provided by others while providing it insufficiently herself.

4 Lindahl Equilibrium

Now it is time for us to go back to our general equilibrium framework. Recall our "intuitive definition" of externality in Section 2: an externality is a case in which the economic activities of one party–a consumer or a firm–have a direct impact on the preferences or production sets of others. To modify our standard model,[7] we need to assume that, the utility of consumer i depends not only on her own consumption bundle x_i, but on the entire vector (x, y) of consumption and production in this economy.

4.1 Why the First Welfare Theorem Fails

Like in the simple examples in Section 2, when externalities are present, a Walrasian equilibrium allocation need not to be Pareto optimal. A natural question is, why would not the proof of the first welfare theorem work in this case?

Recall the proof we discussed in class. To start, we take a Walrasian equilibrium (p^*, x^*, y^*) with $p^* \geq 0$, and an alternative allocation (x', y') that Pareto dominates (x^*, y^*).

[6] If the public good is undesirable, it would be over-provided.

[7] For consistency and simplicity, we do not consider production externalities in this model–consumption externalities only.

Clearly, (x', y') must be feasible, that is,

$$\sum_{i=1}^{I} x_i' \leq \sum_{i=1}^{I} \omega_i + \sum_{j=1}^{J} y_j';$$

and since $p^* \geq 0$,

$$\sum_{i=1}^{I} p^* \cdot x_i' \leq \sum_{i=1}^{I} p^* \cdot \omega_i + \sum_{j=1}^{J} p^* \cdot y_j' \leq \sum_{i=1}^{I} p^* \cdot \omega_i + \sum_{j=1}^{J} p^* \cdot y_j^*,$$

where the second inequality holds because y^* is a profit-maximizing production plan (which follows from the definition of Walrasian equilibrium), so $p^* \cdot y_j^* \geq p^* \cdot y_j'$ for all $j = 1, \ldots, J$. The proof is good up to this point.

In the next step of the proof, we would like to show the following assertion is true: for any consumer i, $x_i' \succsim_i x_i^*$ implies $p^* \cdot x_i' \geq p^* \cdot x_i^*$, and $x_i' >_i x_i^*$ implies $p^* \cdot x_i' > p^* \cdot x_i^*$. But in our modified model, consumer i 's preference relation are not defined on her consumption set X_i; instead, it is defined on $X \times Y$.[8] Consequently, it makes no sense for us to write $x_i' >_i x_i^*$; we should write $(x', y') \succsim_i (x^*, y^*)$ instead.

Since externalities are present, it is possible that $(x', y') \succeq_i (x^*, y^*)$, but $x_i' < x_i^*$ for some consumer i: (x', y') gives her strictly less *direct* consumption, hence it is certainly affordable, but consumer i still prefers (x', y') to (x^*, y^*) because it is associated with lower level of externality-generating activities carried out by other consumers or firms. For instance, it may call for less consumption of some noxious good by a neighbor, or less production, and hence less pollution by some neighboring plant. Therefore, we are not able to prove the assertion, and the proof no longer works.

4.2 Lindahl Equilibrium and a Modified "First Welfare Theorem"

At the level of general equilibrium, Lindahl equilibrium, named after Lindahl (1958), has a flavor of the Coase theorem we discussed in Section 2.2. In a Lindahl equilibrium, transfers are made between every pair of agents for every activity undertaken by one that might have an external effect on the other. Specifically, in our current setting, we have

(1) prices $p \in \mathbb{R}^L$ for the goods themselves;

(2) for every pair of consumers, say i and i', a set of transfer prices $r_i^{i'} \in \mathbb{R}^L$ that records transfers from i to i' made for the consumption choice of x_i; and

[8] X is defined as the product set of all consumers' consumption sets: $X = \Pi_{i=1}^{I} X_i$; and Y is similarly defined: $Y = \Pi_{j=1}^{J} Y_j$, where Y_j is the production set of firm j

(3) for every firm j and consumer i, a set of transfer prices $t_j^i \in \mathbb{R}^L$ that records transfers from j to i made for firm j 's choice of production plan, y_j.

Given prices and a production set Y_j, firm j chooses production plan to solve the following problem:

$$\max_{y_j \in Y_j} p \cdot y_j - \sum_{i=1}^{I} t_j^i \cdot y_j;$$

that is, firm j maximize the transfer-included profits. To work on the consumers' side, we firstly define the "new budget set" of consumer i by

$$\Gamma_i(p^*, r^*, t^*, \omega_i) =$$
$$\left\{(x, y) : p^* \cdot x_i + \sum_{i' \neq i} r_{i'}^{i*} \cdot x_i \leq p^* \cdot \omega_i + \sum_{j=1}^{J} \theta_{ij} \left(p^* \cdot y_j - \sum_{i'=1}^{I} t_j^{i'*} \cdot y_j\right) + \sum_{j=1}^{J} t_j^{i*} \cdot y_j + \sum_{i' \neq i} r_i^{i'*} \cdot x'_{i'}\right\}.$$

So consumer i not only pays for the goods she consumes, but also transfers to other customers for her consumption choice; she also receives transfers from other consumers and firms for their consumption and production choices, respectively. Accordingly, her demand correspondence is

$$D_i(p^*, r^*, t^*, \omega_i) = \{(x', y') \in \Gamma_i(p^*, r^*, t^*, \omega_i) : (x', y') \succsim_i (x, y) \text{ for all } (x, y) \in \Gamma_i(p^*, r^*, t^*, \omega_i)\}.$$

A Lindahl equilibrium can be formally defined as follows:

Definition 1 (Lindahl equilibrium)**.** A vector $(p^*, t^*, r^*, x^*, y^*)$ is a Lindahl equilibrium if

1. (Price-taking profit maximization) for $j = 1, \ldots, J$, $y_j^* \in Y_j$, and $p^* \cdot y_j^* - \sum_{i=1}^{I} t_j^{i*} \cdot y_j^* \geq p^* \cdot y_j - \sum_{i=1}^{I} t_j^{i*} \cdot y_j$ for all $y_j \in Y_j$;

2. (Price-taking preference maximization) for $i = 1, \ldots, I$, $(x^*, y^*) \in D_i(p^*, r^*, t^*, \omega_i)$;

3. (Market clearing) $\sum_{i=1}^{l} x_i^* = \sum_{i=1}^{\prime} \omega_i + \sum_{j=1}^{J} y_j^*$, that is, (x^*, y^*) is a feasible allocation.

Remark 1. *It is important to note that, every consumer chooses the full vector (x^*, y^*), and it is an equilibrium conditions that these choices are identical, and in particular, coincides with firms' choice on production.*

The next theorem is the "first welfare theorem" for Lindahl equilibrium. It shows that, even with externalities in consumption, a Lindahl equilibrium is Pareto optimal.

Theorem 1. *If $(p^*, t^*, r^*, x^*, y^*)$ is a Lindahl equilibrium with $p^* \geq 0$, and preferences are locally nonsatiated, then (x^*, y^*) is Pareto optimal.*

For a proof of Theorem 1, see Kreps (2013), page 382. Very loosely speaking, a Lindahl equilibrium is Pareto optimal since, by imposing transfers, it forces each party in the economy to take other agents' well-being into account. The exposition above is very general—although it equips you a better idea on the big picture, concreteness is, inevitably, sacrificed. To this end, you may want to see Mas-Collel et al. (1995), Sections 11.C and 16.G (in particular, Example 16.G.3), for specific examples in partial equilibrium and general equilibrium, respectively.

These examples focus on the application of Lindahl equilibrium in the context of public goods. Heuristically, when there is only one public good, the equilibrium concept reduces to the following: assign personalized prices, which is determined by each consumer's preference and wealth/endowment, of the public good to all consumers. In this case, because there is only one good that generates externality, the so called "personalized price" is just the sum of price for the good itself and all transfer prices; and in a Lindahl equilibrium, this set of personalized prices guarantees that all consumers choose the same amount of public good consumption, and equals to the efficient level, say q^o. As Ted Bergstrom puts, "With private goods, different people can consume different quantities, but in equilibrium they all must pay the same prices. With public goods, everyone must consume the same amount quantity, but in Lindahl equilibrium, they may pay different prices."

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