# Withholding Verifiable Information
# Supplementary Appendix (For Online Publication)

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## Contents



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# B Supplementary Results for Ternary Actions

In this section, we study the special case where the receiver has three actions: $N = \{1, 2, 3\}$. Recall that $\{\gamma_L, \gamma_H\}$ is feasible for the interval $[\underline{\omega}, \overline{\omega}]$ if there exists a mean preserving contraction of $F|_{[\underline{\omega}, \overline{\omega}]}$ whose support is $\{\gamma_L, \gamma_H\}$.

**Claim B.1.** *If there does not exist $y \in [0, \gamma_2]$ such that $\{\gamma_2, \gamma_3\}$ is feasible for $[y, 1]$, then every bi-pooling solution is implementable.*

*Proof of Claim B.1.* By Lemma 6, $\{\gamma_2, \gamma_3\}$ is feasible for the interval $[y, 1]$ if and only if

(i) $y \leq \gamma_2 \leq m(y) \leq \gamma_3 \leq 1$, and

(ii) $\mathbb{E}[\omega \,|\, \omega \in [\eta(\gamma_2; y), 1]] \geq \gamma_3$,

where $m(y) := \mathbb{E}[\omega \,|\, \omega \in [y, 1]]$, and $\eta(\gamma_2; y)$ is such that $\mathbb{E}[\omega \,|\, \omega \in [y, \eta(\gamma_2)] = \gamma_2$. Then if there does not exist $y \in [0, \gamma_2]$ such that $\{\gamma_2, \gamma_3\}$ is feasible for $[y, 1]$, there are two cases:

(a) (i) fails to hold for all $y \in [0, \gamma_2]$;

(b) (i) holds for some $y \in [0, \gamma_2]$, but (ii) fails for all such $y$'s.

For Case (a), the only possibility is that $m(y) \geq \gamma_3$. If this is the case, every bi-pooling solution $G$ has $\text{supp}\,(G) \subseteq [\gamma_3, 1]$, and hence the unique bi-pooling partition has $B_3 = [0, 1]$, and revelation proofness holds.

For Case (b), let us introduce some notation first. For each $i = 1, 2$, if there exists $h \geq 0$ such that $\mathbb{E}[\omega \,|\, \omega \in [h, 1]] = \gamma_i$, set $h(\gamma_i) = h$; otherwise let $h(\gamma_i) = 0$. Then for every $y \in [0, \gamma_2] \cap [h(\gamma_2), h(\gamma_3)]$, (i) holds. If (ii) fails for all such $y$'s, it must be that $\mathbb{E}[\omega \,|\, \omega \in [\eta(\gamma_2; h(\gamma_2)), 1]] < \gamma_3$. Note that this is not possible if $h(\gamma_2) > 0$: if this is the case, $\eta(\gamma_2; h(\gamma_2)) = 1$ by definition, so it must be that $\mathbb{E}[\omega \,|\, \omega \in [\eta(\gamma_2; h(\gamma_2)), 1]] \geq \gamma_3$, a contradiction. Consequently, in Case (b), (ii) must fail for $y = 0$, and hence every bi-pooling solution $G$ has $G(\gamma_3) - G(\gamma_3^-) = 1 - F(h(\gamma_3))$, and $\text{supp}\,(G) \subseteq [\gamma_2, \gamma_3]$. Thus, the unique bi-pooling partition associated with $G$ has $B_2 = [0, h(\gamma_3)]$ and $B_3 = [h(\gamma_3), 1]$. Since $h(\gamma_3) < \gamma_3$ by definition, revelation proofness must hold. This completes the proof. ■

**Claim B.2.** *All bi-pooling solutions to the information design problem are associated with the same bi-pooling partition.*

*Proof of Claim B.2.* It can be readily seen from the proof of Claim B.1 that, if there does not exist $y \in [0, \gamma_2]$ such that $\{\gamma_2, \gamma_3\}$ is feasible for $[y, 1]$, then all bi-pooling solutions are associated with the same bi-pooling partition. Now suppose there exists $y \in [0, \gamma_2]$ such that $\{\gamma_2, \gamma_3\}$ is feasible for $[y, 1]$. Let $Y$ denote the set of such $y$'s; Lemma 6 implies that $Y$ is a closed subset of $[0, \gamma_2]$. Consequently, the commitment payoff can be identified by the lower endpoint of the bi-pooling interval. Hence, each of them corresponds to a point in $Y$ that maximizes the sender's ex-ante payoff (recall that $u_1 = 0$):

$$\pi(y) = (1 - F(y)) \left[ \frac{\gamma_3 - m(y)}{\gamma_3 - \gamma_2} u_2 + \frac{m(y) - \gamma_2}{\gamma_3 - \gamma_2} u_3 \right].$$

Taking derivative,

$$\pi'(y) = -f(y) \left[ \frac{\gamma_3 - m(y)}{\gamma_3 - r_1} u_2 + \frac{m(y) - \gamma_2}{\gamma_3 - \gamma_2} u_3 \right] + (1 - F(y)) \frac{u_3 - u_2}{\gamma_3 - u_2} m'(y),$$

where

$$m'(y) = \frac{(m(y) - y) f(y)}{1 - F(y)}.$$

Consequently,

$$\begin{aligned} \pi'(y) &= \frac{f(y)}{\gamma_3 - \gamma_2} \left[ (m(y) - y)(u_3 - u_2) - (\gamma_3 - m(y)) u_2 - (m(y) - \gamma_2) u_3 \right], \\ &= \frac{f(y)}{\gamma_3 - \gamma_2} \left[ u_3 (\gamma_2 - y) - u_2 (\gamma_3 - y) \right] \end{aligned}$$

and its sign is determined by the terms between the squared brackets, which implies that $\pi$ is single-peaked in $y$. Therefore, there must exist a unique $z$ that maximizes $\pi(y)$ on $Y$. As a consequence, all bi-pooling solutions are associated with the same bi-pooling partition $\mathcal{B}$ with $B_1 = [0, z]$, $B_2 = [\underline{b}_2, \bar{b}_2]$, and $B_3 = [z, \underline{b}_2] \cup [\bar{b}_2, 1]$. ■

**Claim B.3.** *If no commitment solution is implementable, the sender-preferred laminar PE $\mathcal{B}$ is such that $B_1 = [0, y]$, $B_2 = [h, b]$, and $B_3 = [y, h] \cup [b, 1]$, where $\gamma_2 < b \leq \gamma_3$, and $h > 0$ and $y \geq 0$ are defined by*

$$\mathbb{E}\left[\omega \mid \omega \in [h, b]\right] = \gamma_2 \quad \textit{and} \quad \mathbb{E}\left[\omega \mid \omega \in [y, h] \cup [b, 1]\right] = \gamma_3. \tag{B.1}$$

*Furthermore,*

$$u_3 \leq \frac{\gamma_3 - y}{\gamma_2 - y} u_2. \tag{B.2}$$

*Proof of Claim B.3.* By Theorem 2, if $B_2$ has nonempty interior, it must be that $\mathbb{E}[\omega \mid$

$\omega \in B_2] = \gamma_2$, and $\mathbb{E}[\omega \mid \omega \in B_3] = \gamma_3$.[1] Then to obtain the statement it suffices to show three things: (1) $b > \gamma_2$, namely $B_2$ has nonempty interior; (2) there exists such an $h$; and (3) there exists such a $y$.

We show that $B_2$ has nonempty interior first. Suppose to the contrary that $\text{int}(B_2) = \varnothing$, then by Claim 7, there are two cases: $B_3 = [0,1]$, and $B_3 = [z,1]$ for some $z > 0$. If $B_3 = [0,1]$, since the sender attains highest possible payoff in equilibrium, it must be that a commitment outcome is implementable, a contradiction. If instead $B_3 = [z,1]$ for some $z > 0$, it must be that $z \le \gamma_2$, as otherwise recommending action 2 on $[z, \gamma_2]$ is a profitable deviation. Consequently, this bi-pooling partition is revelation-proof, and hence a commitment outcome is implementable, again a contradiction. Therefore, it must be that $B_2$ has nonempty interior.

To see that there exists such an $h$, we first claim that $\mathbb{E}[\omega \mid \omega \in [0, \gamma_3]] \le \gamma_2$. Suppose not, so

$$\mathbb{E}[\omega \mid \omega \in [0, \gamma_3]] > \gamma_2. \tag{B.3}$$

Without loss of generality, assume that there exists a bi-pooling solution features $[y, 1]$ bi-pooled to $\{\gamma_2, \gamma_3\}$ for some $y \in [0, \gamma_2]$.[2] Consequently, there exist $\bar{b}_1$ and $\underline{b}_1$ with $y \le \underline{b}_1 \le \bar{b}_1$ such that the (unique) bi-pooling partition associated with the bi-pooling solution $\mathcal{B}$ is given by $B_1 = [0, y]$, $B_2 = [\underline{b}_1, \bar{b}_1]$, and $B_3 = [y, \underline{b}_1] \cup [\bar{b}_1, 1]$. Then because $\mathbb{E}[\omega \mid \omega \in B_2] = \gamma_2$ and $\underline{b}_1 \ge 0$, we must have $\bar{b}_1 \le \gamma_3$ by (B.3). Thus, the bi-pooling solution must be implementable, a contradiction. As a consequence, it must be that $\mathbb{E}\left[\omega \mid \omega \in [h, b]\right] = \gamma_2$; it remains to show that $h > 0$. If instead $h = 0$, then $\mathbb{E}[\omega \mid \omega \in [0, \gamma_3]] = \gamma_2$. Consequently, it must be that $B_2 = [0, \gamma_3]$ and $B_3 = [\gamma_3, 1]$. This cannot be optimal: for any $\varepsilon \in (0, \gamma_2)$, define $\hat{B}_1 = [\varepsilon, \gamma_3]$, and $\hat{B}_2 = [0, \varepsilon] \cup [\gamma_3, 1]$. Then for $\varepsilon$ small enough, $\mathbb{E}[\omega \mid \omega \in \hat{B}_i] \ge \gamma_i$ for each $i = 1, 2$, and the sender's ex-ante payoff is strictly higher. Thus, it must be that $h > 0$.

To show that there exists such a $y$, it suffices to show that $\mathbb{E}[\omega \mid \omega \in [0, h] \cup [b, 1]] \le \gamma_3$. Suppose not, so $\mathbb{E}[\omega \mid \omega \in [0, h] \cup [b, 1]] > \gamma_3$. Let $\delta > 0$ be small enough, and let $\epsilon(\delta)$ be such that

$$\mathbb{E}\left[\omega \mid \omega \in [h + \varepsilon(\delta), b - \delta]\right] = \gamma_2.$$

Now define $\tilde{B}_1 = [h + \varepsilon(\delta), b - \delta]$, and $\tilde{B}_2 = [0, h + \varepsilon(\delta)] \cup [b - \delta, 1]]$. Because the density

[1] $B_3$ must have nonempty interior, or else revelation proofness cannot be satisfied. Also by revelation proofness, $b \le \gamma_3$.

[2] If such $y$ does not exist, by Claim B.1, the bi-pooling solution must be implementable, a contradiction.

$f$ is strictly positive, for small enough $\delta$, $\mathbb{E}[\omega \mid \omega \in \tilde{B}_2] \geq \gamma_3$, and $\mu_F(\tilde{B}_2) > \mu_F(B_3)$. This creates a profitable deviation to the sender without violating revelation proofness.

Finally, to show that (B.2) must hold, suppose to the contrary that

$$u_3 > \frac{\gamma_3 - y}{\gamma_2 - y} u_2.$$

An argument analogous to Case 1 (II) in the proof of Proposition 2 shows that the sender has a profitable deviation, and hence the bi-pooling partition $\mathcal{B}$ with $B_1 = [0, y]$, $B_2 = [h, b]$, and $B_3 = [y, h] \cup [b, 1]$ cannot be associated with a sender-preferred laminar PE, a contradiction. ■

By Claim B.3, the sender's ex-ante payoff in a sender-preferred equilibrium can be written as

$$\bar{V}(b) = u_2[F(b) - F(h(b))] + u_3[1 - F(b) + F(h(b)) - F(y(b))],$$

where $h(b)$ and $y(b)$ are implicitly defined by the two equations in (B.1).

**Claim B.4.** *If no commitment outcome is implementable, the sender's ex-ante payoff in a sender-preferred equilibrium, $\bar{V}(b)$, is increasing in $b$.*

*Proof of Claim B.4.* Directly,

$$\bar{V}'(b) = (u_2 - u_3) \left[ f(b) - f(h) \frac{\mathrm{d}h}{\mathrm{d}b} \right] - u_3 f(y) \frac{\mathrm{d}y}{\mathrm{d}b}. \tag{B.4}$$

Using (B.1), by the implicit function theorem,

$$\frac{\mathrm{d}h}{\mathrm{d}b} = -\frac{(b - \gamma_2) f(b)}{(\gamma_2 - h) f(h)}, \tag{B.5}$$

$$\frac{\mathrm{d}y}{\mathrm{d}b} = -\frac{(b - h)(\gamma_3 - \gamma_2) f(b)}{(\gamma_2 - h)(\gamma_3 - y) f(y)}. \tag{B.6}$$

Plugging (B.5) and (B.6) into (B.4),

$$\begin{aligned}\bar{V}'(b) &= (u_2 - u_3)\, f(b) \left(1 + \frac{b-\gamma_2}{\gamma_2 - h}\right) + u_3 f(b) \frac{(b-h)\,(\gamma_3-\gamma_2)}{(\gamma_2-h)\,(\gamma_3-y)} \\ &= \left[(u_2-u_3)\,\frac{b-h}{\gamma_2-h} + u_3\frac{(\gamma_3-\gamma_2)\,(b-h)}{(\gamma_2-h)\,(\gamma_3-y)}\right] f\,(b) \\ &= \left[u_2\frac{b-h}{\gamma_2-h} - u_3\frac{(b-h)(\gamma_2-y)}{(\gamma_2-h)(\gamma_3-y)}\right] f\,(b) \\ &= \left[u_2(\gamma_3-y) - u_3(\gamma_2-y)\right] \frac{(b-h)f\,(b)}{(\gamma_2-h)(\gamma_3-y)},\end{aligned}$$

and we see that $\bar{V}'(b) \geq 0$ if and only if $u_2(\gamma_3 - y) \geq u_3(\gamma_2 - y)$. Then since no commitment solution is implementable, by Claim B.3, (B.2) implies that $\bar{V}'(b) \geq 0$, and hence the sender's ex-ante payoff in a sender-preferred equilibrium is increasing in $b$. ■

# C Remaining Omitted Proofs

## C.1 Proof of Claim 1

To solve for a sender-preferred equilibrium, we find a bi-pooling solution to the corresponding information design problem first, and check whether it is implementable using Corollary 1. If it is, a sender-preferred equilibrium is associated with a barely obedient bi-pooling partition $\mathcal{B}$ that is also associated with the commitment solution.[3]

Now suppose that no commitment solution is implementable. By Theorem 2, there exist $y$, $h$, and $b$ such that $B_2 = [h, b]$, and $B_3 = [y, h] \cup [b, 1]$.[4] By Claim B.3, there exist $y \geq 0$ and $h > 0$ such that

$$\mathbb{E}\left[\omega \mid \omega \in [h, b]\right] = \gamma_2 \quad \text{and} \quad \mathbb{E}\left[\omega \mid \omega \in [y, h] \cup [b, 1]\right] = \gamma_3.$$

Consequently, $y$ and $h$ can be implicitly defined as functions of $b$, and hence the sender's

[3] One may wonder what if the bi-pooling solution found above is not implementable, but there exists another bi-pooling solution that is implementable. This can never happen when there are three actions: by Claim B.1, all bi-pooling solutions induce essentially the same bi-pooling partition.

[4] If both $B_2$ and $B_3$ are intervals, just set $y = h$.

ex-ante payoff can be parametrized by $b$, so long as $b \leq \gamma_3$:

$$\bar{V}(b) = u_2[F(b) - F(h(b))] + u_3[1 - F(b) + F(h(b)) - F(y(b))].$$

By Claim B.4, $\bar{V}$ is increasing in $b$. Hence, the partition corresponding to the sender's preferred equilibrium can be found by setting $b = \gamma_3$, which yields the expression in the statement of the claim.

## C.2 Proof of Corollary 2

Recall that $\gamma_1 := 0$ and $u_0$ is normalized to zero; hence when $n = 3$, Equation (4) in the main text reduces to

$$\frac{u_3 - u_2}{\gamma_3 - \gamma_2} > \frac{u_2}{\gamma_2 - \max\{0, h(\gamma_2; \gamma_3)\}} \tag{C.1}$$

And because $h(\gamma_2; \gamma_3) \geq 0$, the right-hand side of (C.1) further reduces to $u_2/(\gamma_2 - h(\gamma_2; \gamma_3))$. Then since $f$ is increasing, $\gamma_3 - \gamma_2 \geq \gamma_2 - h(\gamma_2; \gamma_3)$; thus, if $u_3 - u_2 > u_2$, or $u_3 > 2u_2$, (C.1) must hold. Consequently, by Proposition 2, every bi-pooling solution can be implemented. By Claim B.1, all bi-pooling solutions induce the same bi-pooling partition. Then because the set of bi-pooling solutions is the set of extreme points of the solution correspondence of the information design problem, all commitment solutions must be associated with the same bi-pooling partition. Thus, every commitment outcome is implementable.

## C.3 Proof of Claim 2

Let $\omega_q$ denote the cutoff quality that the buyer is indifferent between purchasing $q-1$ and $q$ units: it solves

$$\omega_q U(q) - pq = \omega_q U(q-1) - p(q-1),$$

so $\omega_q = p/[U(q) - U(q-1)]$. Letting $\omega_0 = 0$ and $\omega_{n+1} = 1$, the buyer buys $q \in \{0, 1, \ldots, n\}$ units of the product if and only if $\omega \in [\omega_q, \omega_{q+1}]$. If $P > 2A$, $U''(q)/[U'(q)]^2$ is strictly decreasing in $q$, and thus

$$\omega_{q+1} - \omega_q = \frac{p}{U(q+1) - U(q)} - \frac{p}{U(q) - U(q-1)}$$

is strictly decreasing in $q$. By noting that $v_q - v_{q-1} = p - c$ for all $q \in \{0, 1, \ldots, n\}$, Claim 2 is implied by Corollary 2.

## C.4 Proof of Claim 3

If either $\alpha_b^j$ increases for all $j = 1, \ldots, N$, or $\beta_b^j$ increases for all $j = 1, \ldots, N$, or both, $\gamma_2^j$ decreases for every $j$, and hence $\gamma_j^m$ decreases. By Claim 1, when no commitment outcome can be implemented, the sender's ex-ante payoff is given by

$$\bar{V}(\gamma_3^m) = u_2[F(\gamma_3^m) - F(h(\gamma_3^m))] + u_3[1 - F(\gamma_3^m) + F(h(\gamma_3^m)) - F(y(\gamma_3^m))],$$

where $h$ and $y$ are implicitly defined by

$$\mathbb{E}\left[\omega \mid \omega \in [h, \gamma_3^m]\right] = \gamma_2^m \quad \text{and} \quad \mathbb{E}\left[\omega \mid \omega \in [y, h] \cup [\gamma_3^m, 1]\right] = \gamma_3^m.$$

Now

$$\bar{V}'(\gamma_3^m) = (u_2 - u_3)\left[f\left(\gamma_3^m\right) - f(h)\frac{\mathrm{d}h}{\mathrm{d}\gamma_3^m}\right] - u_3 f(y)\frac{\mathrm{d}y}{\mathrm{d}\gamma_3^m}. \tag{C.2}$$

By the implicit function theorem,

$$\frac{\mathrm{d}h}{\mathrm{d}\gamma_3^m} = -\frac{\left(\gamma_3^m - \gamma_2^m\right) f\left(\gamma_3^m\right)}{\left(\gamma_2^m - h\right) f(h)}, \tag{C.3}$$

$$\frac{\mathrm{d}y}{\mathrm{d}\gamma_3^m} = \frac{\left(\gamma_2^m - h\right)\left[1 - f\left(\gamma_3^m\right) + F(h) - F(y)\right]}{\left(\gamma_2^m - h\right)\left(\gamma_3^m - y\right) f(y)} - \frac{\left(\gamma_3^m - h\right)\left(\gamma_3^m - \gamma_2^m\right) f\left(\gamma_3^m\right)}{\left(\gamma_2^m - h\right)\left(\gamma_3^m - y\right) f(y)}. \tag{C.4}$$

Plug (C.3) and (C.4) into (C.2),

$$\begin{aligned}\bar{V}'(\gamma_3^m) = &-\frac{\left(\gamma_2^m - h\right)\left[1 - f\left(\gamma_3^m\right) + F(h) - F(y)\right] u_3}{\left(\gamma_2^m - h\right)\left(\gamma_3^m - y\right)} - \frac{\left(\gamma_3^m - h\right)\left(\gamma_2^m - y\right) f\left(\gamma_3^m\right)}{\left(\gamma_2^m - h\right)\left(\gamma_3^m - y\right)} u_3 \\ &+ \frac{\left(\gamma_3^m - h\right)\left(\gamma_3^m - y\right) f\left(\gamma_3^m\right)}{\left(\gamma_2^m - h\right)\left(\gamma_3^m - y\right)} u_2,\end{aligned}$$

whose sign is determined by

$$-\left(\gamma_2^m - h\right)\left(1 - F\left(\gamma_2^m\right) + F(h) - F(y)\right) - \left(\gamma_3^m - h\right) f\left(\gamma_3^m\right)\left[\left(\gamma_2^m - y\right) u_3 - \left(\gamma_2^m - y\right) u_2\right]. \tag{C.5}$$

By Claim B.3, if no comment outcome is implementable, it must be that $(\gamma_2^m - y)u_3 \leq (\gamma_3^m - y)u_2$. Consequently, the sign of the second term of (C.5) must be positive, and the first term has a strictly negative sign. Hence as $\gamma_3^m$ decreases, the expert's ex-ante payoff in her preferred equilibrium strictly decreases if the second term is larger in absolute value, which establishes the statement.

# D Equilibrium Refinement

In this section, we show that any partitional equilibrium (PE) defined in Section 3 survives the Never-a-Weak-Best-Response (NWBR) criterion proposed by Cho and Kreps (1987), and is a Grossman-Perry-Farrell equilibrium (Bertomeu and Cianciaruso, 2018). We use the term "type" instead of "state" henceforth to ease exposition.

## D.1 Never-a-Weak-Best-Response (NWBR) Criterion

We introduce some notation first. For any $m \in \mathcal{C}$, let $MBR(m)$ denote the set of all mixed strategy best responses for the receiver to message $m$ for any belief $p(\cdot \mid m)$.[5] Moreover, let $v^*_\omega$ denote the equilibrium payoff of type $\omega$. Finally, for any equilibrium and an off-path message $m$, define

$$D(\omega, m) = \left\{ \rho \in MBR(m) : v^*_\omega < \sum_{i \in N} u_i \rho_i \right\},$$

and

$$D^0(\omega, m) = \left\{ \rho \in MBR(m) : v^*_\omega = \sum_{i \in N} u_i \rho_i \right\};$$

in words, $D(\omega, m)$ is the set of mixed strategy best responses that make type $\omega$ strictly prefer $m$ to her equilibrium message, and $D^0(\omega, m)$ is the set of mixed strategy best responses that make type $\omega$ exactly indifferent.

**Definition 1.** An equilibrium $(\sigma, \tau, p)$ **survives the NWBR criterion** if for every $m \in \mathcal{C}$ and any $\omega, \omega' \in [0, 1]$, $D^0(\omega', m) \subseteq \cup_{\omega \neq \omega'} D(\omega, m)$ implies that $\omega' \notin \operatorname{supp} p\,(\cdot \mid m)$.

**Claim D.1.** *For every PE $(\sigma, \tau, p)$, there exists $p'$ such that $(\sigma, \tau, p')$ survives the NWBR criterion.*

*Proof.* Fix a PE $(\sigma, \tau, p)$, and let $\mathcal{B}$ denote the associated partition. For every $m \notin \mathcal{B}$, let $\ell = \min\{i : m \cap B_i \neq \varnothing\}$. Let $p'$ be such that $p'(\cdot \mid B_i) = p(\cdot \mid B_i)$ for all

[5] Note that $p$ must satisfy $\operatorname{supp} p(\cdot \mid m) \subseteq m$.

$i = 0, \ldots, n-1$, and for any $m \notin \mathcal{B}$, let

$$p'\left(\min m \cap B_\ell \mid m\right) = 1. \tag{D.1}$$

By the definition of PE, the receiver never mixes on path, and hence for any $\omega' \in [0,1]$, $D^0(\omega', m) = \{\delta_k\}$ if and only if $\omega \in B_k \backslash \left(\cup_{i>k} B_i\right)$, where $\delta_k$ is the Dirac measure at action $k$. Furthermore, define

$$M = \{k \in N : k \text{ is such that } m \cap B_k \neq \varnothing\};$$

then the lowest action in $M$ is $\ell$. Now for any $\omega' \in [0,1]$,

$$\bigcup_{\omega \neq \omega'} D(\omega, m) = \{\rho \in \Delta(M) : \operatorname{supp} \rho \subseteq \{a_k, a_{k+1}\} \text{ with } k \geq \ell, \text{ and } \rho(a_\ell) < 1\}.$$

Then $D^0(\omega', m) \subseteq \cup_{\omega \neq \omega'} D(\omega, m)$ if and only if $D^0(\omega', m) = \{\delta_k\}$ with $k > \ell$, which is in turn equivalent to $\omega' \in \cup_{k>\ell}\left(B_k \backslash \left(\cup_{i>k} B_i\right)\right)$. Then (D.1) implies that $\omega' \notin \operatorname{supp} p'(\cdot \mid m)$. Consequently, $(\sigma, \tau, p')$ survives the NWBR criterion. ■

## D.2 Grossman-Perry-Farrell Equilibrium

**Definition 2.** Fix a PE $(\sigma, \tau, p)$. Say that $m^* \in \mathcal{C}$ is a **self-signaling set** if[6]

$$m^* = \left\{\omega \in m^* : v\left(\mathbb{E}[\omega \,|\, \omega \in m^*]\right) > v\left(\mathbb{E}[\omega \,|\, \omega \in \sigma(\omega)]\right)\right\}.$$

An PE is a **Grossman-Perry-Farrell equilibrium** if there does not exist a self-signaling set.

**Claim D.2.** *Every PE is a Grossman-Perry-Farrell equilibrium.*

*Proof.* Fix a PE $(\sigma, \tau, p)$, and let $\mathcal{B}$ denote the associated partition. Suppose there exists a self-signaling set $m^*$. Let

$$\bar{k} := \max\left\{i \in N : \text{there exists } \omega \in m^* \text{ such that } \sigma(\omega) = B_i\right\}.$$

Because $m^*$ is a self-signaling set, it must be that $\mathbb{E}[\omega \,|\, \omega \in m^*] \geq \gamma_{\bar{k}+1}$. But then

[6] We could have defined a self-signaling set for any equilibrium, but doing that largely complicates the notation: here, $\sigma(\cdot)$ is a well-defined function that maps a state to a subset of the state space because $(\sigma, \tau, p)$ is an ORE.

revelation proofness of $\mathcal{B}$ implies that there must exist $\omega' \in m^*$ such that $\sigma(\omega') = B_j$ with $j \geq \bar{k} + 1$, a contradiction. ■

# References


Bertomeu, J. and D. Cianciaruso (2018): "Verifiable disclosure," *Economic Theory*, 65, 1011–1044.

Cho, I.-K. and D. M. Kreps (1987): "Signaling Games and Stable Equilibria," *The Quarterly Journal of Economics*, 102, 179–221.