Uncharted Waters: Selling a New Product Robustly Supplementary Appendix (For Online Publication)

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Appendix C Detailed Discussion of Model Assumptions

In Section C.1, I allow Seller to recognize whether Buyer is a first-time visitor or came back from search. Section C.2 shows that the main insights of this paper are qualitatively robust under various alternative assumptions.

C.1 Recognizable Buyer Identity

C.1.1 Exploding Offers and Renegotiation

One way that Seller can take advantage of this is to make an exploding offer: she commits not to sell to Buyer if she does not buy during her first visit. In this case, Buyer buys without search if and only if $w - p \geq \xi - s$, namely when her value of Seller's product is no less than the expected value of the unknown outside option net of the search cost; and if she goes to search, she would never come back. The probability of this event is $1 - H((p + \xi - s)^-)$, and hence Seller's revenue guarantee from an exploding offer is $p[1 - H((p + \xi - s)^-)]$.[1] One striking feature of exploding offers is that Nature's choice of outside option becomes completely irrelevant: it is outcome equivalent to that the outside option distribution is δ_ξ, the degenerate distribution at ξ, and Buyer must incur a cost s to consume the outside option.

Another possibility is that Seller posts a price first, and then if Buyer comes back she may have an incentive to increase the price. As noted in Armstrong and Zhou (2016), in the current framework very little can be said; but if Buyer must incur an exogenous cost $r > 0$ to return to buy Seller's product after search, no matter how small r is, a Diamond paradox (Diamond, 1971) style argument shows that once Buyer goes to search, she would never come back. Hence, the equilibrium outcome is the same as that of exploding offers.

Proposition C.1 summarizes the findings when Buyer's identity is recognizable.

Proposition C.1. *Suppose that Seller can recognize whether Buyer is a first-time visitor. Then*

(i) if Seller can commit to an exploding offer, it is optimal to offer $p = 1 - \xi + s$ with full information;

(ii) for all $\mu, \xi \in (0, 1)$ and $0 \leq s < \xi$, Seller gets a strictly higher revenue guarantee than the case that she cannot distinguish between first-time visitors and searchers.

[1] For a function $f : \mathbb{R} \to \mathbb{R}$, $f(s^-) := \lim_{x \searrow s} f(x)$ whenever this limit exists.

(iii) if Seller cannot commit to the price, and there is a cost of returning to Seller $r > 0$, then the equilibrium outcome is the same as Seller committing to exploding offers.

Proof. I prove part (i) first. For a fixed p, Seller minimizes $H((p+\xi-s)^-)$. There are two cases: $p+\xi-s \le \mu$ and $p+\xi-s > \mu$. If $p+\xi-s \le \mu$, the optimal distribution over posteriors is the degenerate distribution at μ, which corresponds to no information. Consequently, Buyer buys with probability 1, and hence Seller's revenue is exactly p. Thus, it is optimal for Seller to set $p = \mu-\xi+s$ provided the right-hand side is nonnegative, and the revenue guarantee is $\mu - \xi + s$.

Another case is $p + \xi - s > \mu$. In this case, to minimize $H((p+\xi-s)^-)$, it is optimal to put as much mass as possible at $p + \xi - s$, and put the rest of the mass at 0 so that $\mathbb{E}_H[w] = \mu$. Thus, the optimal distribution for a fixed p is the binary distribution with support on $\{0, p + \xi - s\}$. Consequently, Seller's revenue is $p\mu/(p + \xi - s)$, and one can show that this expression is strictly increasing in p. Therefore, the optimal price is $1-\xi+s$, the optimal distribution is the binary distribution with support on $\{0, 1\}$, and the revenue guarantee is $\mu(1 - \xi + s)$.

Finally, note that $\mu(1 - \xi + s) \ge \mu - \xi + s$ and the inequality is strict for all $\mu \in (0, 1)$. Hence, full information, namely the binary distribution with support on $\{0, 1\}$, and price $1 - \xi + s$ is the optimal selling strategy when Seller can commit to an exploding offer.

Part (ii) is obtained by comparing the revenue guarantee in this case, namely $\mu(1 - \xi + s)$, with that identified in the proof of Theorem 1. Finally, Part (iii) follows from an argument similar to the proof of Proposition 4 in Armstrong and Zhou (2016). ∎

The optimality of full information stems from the fact that full information is "efficient" in the sense that it leaves Buyer with no uncertainty on whether she should choose Seller's product or her outside option, and that when the outside option distribution is δ_ξ Seller can appropriate all the surplus simply by pricing at $1-\xi+s$. Recognizable Buyer identity helps Seller because δ_ξ is always suboptimal for Nature when her choice matters.

C.1.2 Price Discrimination

Now I assume that Seller can deviate from the robustly optimal selling strategy in the sense that while the information provision policy cannot be changed, she can commit to a price path (p_1, p_2) with $p_1 < p_2$ such that p_1 and p_2 are the prices charged if Buyer buys

immediately or after search, respectively.[2] In particular, I allow Seller to deviate by either charging a higher price in the second period, or offering a "buy-now discount:" a lower price is offered if Buyer purchases without search, but if she comes back from search she has to pay the equilibrium price.

Proposition C.2. *Suppose that Seller can recognize whether Buyer is a first-time visitor. Let (p_r, H^*) be a robustly optimal selling strategy identified in Theorem 1, and let G^* be the corresponding worst-case outside option distribution. If Seller deviates by committing to a pair of prices (p_1, p_2), where either $p_1 = p_r$ or $p_2 = p_r$, then*

(i) if Nature cannot detect this deviation and hence the outside option distribution is still G^*, Seller can benefit from such a deviation unless H^* corresponds to full information;

(ii) if Nature can detect this deviation and optimally responds to it by choosing a new outside option distribution, Seller cannot benefit from such a deviation.

Proof. For part (i), the outside option distribution is fixed at G^*. If H^* corresponds to full information, that is, it is the binary distribution with mean μ and with support on $\{0, 1\}$. Under full information, there is no buy-later demand and hence charging different prices for "buy-now" and "buy-later" does not increase Seller's payoff. For all other H^*, $1-H^*(p)$ is strictly log-concave on $(p_r, \sup\{\text{supp}(H^*)\})$. Then by Proposition 1 in Armstrong and Zhou (2016), Seller benefits from such a deviation.

Now consider part (ii). Suppose Seller offers prices (p_1, p_2) with $p_1 < p_2$, and upon observing this Nature can choose an outside option distribution different from G^*. Seller's total demand is given by

$$1 - \mathbb{E}_G\left[H^*\left(p_2 + \min\left\{v, S_G^{-1}(s + p_2 - p_1)\right\}\right)\right],$$

and the buy-now demand is $1 - H^*\left(p_2 + S_G^{-1}(s + p_2 - p_1)\right)$. Thus, Seller's buy-later demand is given by

$$\begin{aligned}
&1 - \mathbb{E}_G\left[H^*\left(p_2 + \min\left\{v, S_G^{-1}(s + p_2 - p_1)\right\}\right)\right] - \left[1 - H^*\left(p_2 + S_G^{-1}(s + p_2 - p_1)\right)\right] \\
&= H^*\left(p_2 + S_G^{-1}(s + p_2 - p_1)\right) - \mathbb{E}_G\left[H^*\left(p_2 + \min\left\{v, S_G^{-1}(s + p_2 - p_1)\right\}\right)\right].
\end{aligned}$$

[2] This form of price discrimination is studied in, for example, Nocke, Peitz, and Rosar (2011) and Armstrong and Zhou (2016).

Consequently, Seller's expected payoff is

$$p_1\left[1-H^*\left(p_2+S_G^{-1}(s+p_2-p_1)\right)\right]+$$
$$p_2\left[H^*\left(p_2+S_G^{-1}(s+p_2-p_1)\right)-\mathbb{E}_G\left[H^*\left(p_2+\min\left\{v,S_G^{-1}(s+p_2-p_1)\right\}\right)\right]\right],$$

which is equivalent to

$$p_2\mathbb{E}_G\left[1-H^*\left(p_2+\min\left\{v,S_G^{-1}(s+p_2-p_1)\right\}\right)\right]-(p_2-p_1)\left[1-H^*\left(p_2+S_G^{-1}(s+p_2-p_1)\right)\right].$$

But then since H is affine on $(p_r,1)$, the first term is constant in the choice of G, and the second is minimized by choosing $G=\delta_\xi$. Then the above expression becomes

$$p_2\left[1-H^*\left(p_2+\xi-s-p_2+p_1\right)\right]-(p_2-p_1)\left[1-H^*\left(p_2+\xi-s-p_2+p_1\right)\right]=p_1\left[1-H^*\left(p_1+\xi-s\right)\right].$$

But this implies that Seller cannot benefit from setting different buy-now and buy-later prices. ∎

When Nature cannot detect Seller's deviation and hence the outside option distribution is fixed at G^*, a celebrated result in Armstrong and Zhou (2016) can be used off-the-shelf. They show that whenever the buy-now demand is strictly more elastic than the buy-later demand,[3] charging a higher "buy-later price" benefits Seller because by doing that she either extracts more surplus if Buyer buys after search, or makes Buyer more likely to buy without search. Unless H^* corresponds to full information, in which case there is no buy-later demand and hence such selling tactics would not be useful, the buy-now demand is always strictly more elastic than the buy-later demand.

C.2 Robustness to Alternative Assumptions

C.2.1 Random Prices

If random prices were allowed, many economic insights that emerge from the main model would remain valid. For example, Seller's robustness-seeking motive still begets affinity, although the affine object is the distribution of the buyer's net values of the new product, instead of the distribution over posteriors in the main model. Furthermore, a deterrence

[3] A sufficient condition is that $1-F(p)$ is strictly log-concave, where F is the distribution of Buyer's value of Seller's product. In this model, F is identical to the robustly optimal distribution over posteriors.

policy now entails a mass point in the distribution of net values. In particular, the trade-off between search deterrence and surplus extraction remains: a deterrence policy is only effective when sufficiently high prices are not charged too often.

C.2.2 "Safe" Outside Option

If Buyer has (free) access to a "safe" outside option with value $u_0 > 0$, the seller's expected revenue under $((p, H), G)$ is given by

$$p\,\mathbb{E}_G\left[1 - H\left(p + \max\left\{\min\{a, v\}, u_0\right\}\right)\right].$$

Recall that, as outlined in Appendix A.2.1, in the baseline model uses $z = \min\{a, v\}$ to denote Buyer's effective outside option. I then solve the auxiliary problem in which Nature chooses this effective outside option distribution, and show that Nature's optimal choice can be induced by an outside option distribution.

To solve this new problem, it suffices to define $y := \max\{z, u_0\}$ and work with the distribution of this new variable instead. It can be shown that Nature's optimal distribution of this new variable can be induced by an outside option distribution, and adding this (relevant) "safe" outside option does not change the qualitative features of the main results.

C.2.3 Continuously Distributed Match Value

Suppose instead that Seller and Buyer share a common prior belief that the match value is distributed on $[0, 1]$ according to a CDF F, which admits a strictly positive density f. In this setting, Buyer's expected payoff depends on her posterior belief about the match value solely through its posterior mean. Consequently, Seller's problem of choosing information provision policies reduces to selecting the induced distribution of posterior means. It is well-known that a distribution of posterior means can be induced by an information provision policy if and only if it is a mean-preserving contraction (MPC) of the prior F.[4] Therefore, Seller's revenue guarantee maximization problem is

$$\max_{(p,H)\in[0,1]\times\mathrm{MPC}(F)} \min_{G\in\mathcal{M}(\xi)} p\,\mathbb{E}_G[1 - H(p + \min\{a, v\})], \tag{C.1}$$

[4] See, for example, Blackwell (1953) and Kolotilin (2018).

where MPC(F) is the set of MPCs of F: $H \in \text{MPC}(F)$ if and only if $\int_0^x F(s)\,\mathrm{d}s \leq \int_0^x F(s)\,\mathrm{d}s$ for all $x \in [0,1]$, where the inequality holds with equality at $x = 1$.

This problem is, however, generally intractable. To see why, recall that in the first stage, a continuum of information design problems needs to be solved—one for each price. Even if we further assume that the prior F admits a *single-peaked* density, a full characterization of even *one* of these problems can be infeasible. The difficulty is discussed in detail in Section 5 of Sapiro-Gheiler (2024).[5]

Nonetheless, some key qualitative features of the main results continue to hold when the match value is continuously distributed.

Observation 1. *When the search cost is sufficiently small, Seller does not use a deterrence policy; for larger search costs, a deterrence policy can be optimal. Moreover, the robust price drops when switching to a deterrence policy.*

Since Nature's problem remains unchanged, the key observation that a deterrence policy is effective only when $p \leq s/\xi$ continues to hold. When the search cost is zero, there is no incentive to deter Buyer search, and Seller charges a strictly positive price and gets a strictly positive revenue guarantee. It can be shown that the value of problem (C.1) is continuous in s, implying that for sufficiently small s, deterring search remains unprofitable. This highlights that the trade-off between search deterrence and surplus extraction persists. Furthermore, the reason for adopting a deterrence policy is precisely that it generates higher demand; consequently, at the "switching point," the price must drop, as in the main model.

Moreover, a similar rationale to that in the first observation of Section 3.1 applies here. However, due to the restriction imposed by the MPC constraint, having an affine segment in the distribution of posterior means need not always be optimal.

Observation 2. *Suppose the prior F is either convex or concave. Then for every price, there exist parameter values under which the distribution of posterior means that solves Seller's information design problem has an affine segment.*

C.2.4 Seller's Knowledge about the Outside Option Distribution

The qualitative insights remain unaffected by minor adjustments to upper and lower bounds of the support. To illustrate this, suppose first that the lower bound remains zero

[5] Sapiro-Gheiler's problem is isomorphic to the information design problem when the exogenously given price is zero, and the search cost is also zero; allowing for positive price and search cost makes the problem even more complicated.

and the upper bound can be altered; denote the upper bound by R. Recall from the discussion in the second paragraph of Section 3.1 that Seller can deter search only when the sum of the price and the maximum reservation value, $R - s/\xi$, does not exceed one—that is, $p + R - s/\xi \leq 1$, or equivalently, $p \leq s/\xi + 1 - R$. It is readily seen that a higher upper bound R reduces the profitability of deterring search. Therefore, if the upper bound is either strictly less than one or not substantially greater than one, all main results remain qualitatively intact. However, for any s and ξ with $0 < s < \xi < 1$, there exists $\hat{R}$ such that if the upper bound $R > \hat{R}$, deterring search is never profitable. Similarly, one can show that although lowering the lower bound also diminishes the profitability of deterrence, small changes in either direction do not affect the qualitative conclusions.

An alternative reasonable assumption regarding Seller's knowledge about the outside option distribution is that she knows the mean and an upper bound on a higher moment (for example, variance), but allows the support to be unbounded. Although the robustly optimal distribution over posteriors may become strictly convex rather than affine under this assumption, the qualitative insights persist: deterring search imposes a restriction on price, which generates the trade-off between search deterrence and surplus extraction; moreover, considerable variations in information provision policies continue to emerge.

Appendix D Proof of Proposition 1

D.1 Preliminaries

I proceed as follows: as outlined in Appendix A, for every candidate robustly optimal distribution H^* in Proposition 3, I find a corresponding worst-case effective outside option distribution $\hat{G}^*$, and then use Lemma 1 to show that $(H^*, \hat{G}^*)$ solves problem (10). Next, I show that the worst-case effective outside option distribution $\hat{G}^*$ can be induced by an outside option distribution (or just "induced" for simplicity), which then implies that H^* is indeed a robustly optimal distribution.

To check that an effective outside option distribution can be induced, in some cases I use the following result due to Au and Whitmeyer (2023).

Lemma D.1 (Au and Whitmeyer, 2023)**.** *An effective outside option distribution can be induced if and only if there exists* $\nabla \in \Delta([\xi - s, 1 - s/\xi])$ *where for each* $a \in \text{supp}(\nabla)$ *there*

is a distribution of outside options $G_a \in \mathcal{M}(\xi)$ *such that for each* $z \in [0, 1 - s/\xi]$,

$$\hat{G}(z) = \nabla(z) + \int_{\text{supp}(\nabla)\cap(z,1-s/\xi]} G_a(z)\, \mathrm{d}\nabla(a).$$

Observe also that any price $p > 1 - (\xi - s)$ is weakly dominated by $p = 0$: for any such price, by setting the outside option distribution to be the degenerate distribution at ξ, Nature is able to make Buyer not buy from Seller for sure. This is because the highest posterior that Seller can generate is 1, and hence the highest net value is $1 - p$, but $1 - p < \xi - s$. Consequently, Seller does not sell at all and the revenue guarantee is zero, which is the same as setting $p = 0$. Therefore, when optimizing over p, it suffices to consider $p \in [0, 1 - (\xi - s)]$.

D.2 The Case of $p > s/\xi$

The following two claims establish the results for this case.

Claim D.1. *Suppose* $p > s/\xi$ *and* $\mu > (1 + p)/2$. *Then any distribution over posteriors* $H \in \mathcal{M}(\mu)$ *that first-order stochastically dominates* $U_{[p,1]}$ *is robustly optimal. Furthermore, Seller's revenue guarantee given price* p *is* $\Phi(p) = p[1 - (\xi - s)/(1 - p)]$.

Proof of Claim D.1. Consider any distribution H^* that first-order stochastically dominates $U_{[p,1]}$. Then the concave hull of $H\big|_{[0,1-s/\xi]}(p+z)$ coincides with $U_{[p,1]}$, and Nature can attain the value of problem (11) by using a binary distribution

$$\hat{G}^*(z) = \begin{cases} 1 - \frac{\xi - s}{1 - p} & \text{if } 0 \le z < 1 - p, \\ 1 & \text{if } z \ge 1 - p. \end{cases}$$

Now $\mu > (1 + p)/2$ implies that $p < 2\mu - 1 < \mu$, and hence Seller's value is $\widetilde{G}_p(\mu) = 1 - (\xi - s)/(1 - p)$ for any choice of distribution over posteriors with support on a subset of $[p, 1]$. Thus, $(H^*, \hat{G}^*)$ is a saddle point; and to show that H^* is robustly optimal, it only suffices to show that $\hat{G}^*$ can be induced by some outside option distribution G^*. Consider

$$G^*(v) = \begin{cases} 1 - \frac{\xi - s}{1 - p} & \text{if } 0 \le v < \frac{\xi(1-p)}{\xi - s}, \\ 1 & \text{if } v \ge \frac{\xi(1-p)}{\xi - s}; \end{cases}$$

by the definition of a, $a = 1 - p$. Thus, by definition of the effective outside option

distribution, the effective value distribution induced by G^* is

$$\hat{G}(z) = \begin{cases} G^*(z) & \text{if } 0 \le z < 1-p, \\ 1 & \text{if } z \ge 1-p, \end{cases}$$

which is exactly $\hat{G}^*$. This completes the proof. ∎

In particular, since $\mu > (1+p)/2$, $U_{[2\mu-1,1]}$ is well-defined, has mean μ, and first-order stochastically dominates $U_{[p,1]}$. Consequently, $U_{[2\mu-1,1]}$ is optimal.

Claim D.2. *If $\mu \le (1+p)/2$, there exists $\bar{w} \in [2\mu - p, 1]$ where*

$$\bar{w} = \begin{cases} 2\mu - p & \text{if } \mu > p \text{ and } \xi - s \le \frac{2(\mu-p)^2}{2\mu-p}, \\ \sqrt{(\xi-s)(\xi-s+2p)} + (\xi-s+p) & \text{if } \mu > p \text{ and } \frac{2(\mu-p)^2}{2\mu-p} < \xi - s \le \frac{(1-p)^2}{2}, \\ & \text{or } \mu \le p \text{ and } \xi - s \le \frac{(1-p)^2}{2}, \\ 1 & \text{if } \xi - s \ge \frac{(1-p)^2}{2}, \end{cases}$$

such that the robustly optimal distribution is

$$H_{\bar{w}}(w) = \begin{cases} 1 - \frac{2\mu}{\bar{w}+p} & w \in [0, p), \\ 1 - \frac{2\mu}{\bar{w}+p} + \frac{2\mu}{\bar{w}+p}\left(\frac{w-p}{\bar{w}-p}\right) & w \in [p, \bar{w}), \\ 1 & w \in [\bar{w}, 1]. \end{cases}$$

And Seller's revenue guarantee when the price is p is

$$\Phi(p) = \begin{cases} p\left[1 - \frac{\xi-s}{2(\mu-p)}\right] & \text{if } \mu > p \text{ and } \xi - s \le \frac{2(\mu-p)^2}{2\mu-p}, \\ \frac{p\mu}{\sqrt{(\xi-s)(\xi-s+2p)}+(\xi-s+p)} & \text{if } \mu > p \text{ and } \frac{2(\mu-p)^2}{2\mu-p} < \xi - s \le \frac{(1-p)^2}{2}, \\ & \text{or } \mu \le p \text{ and } \xi - s \le \frac{(1-p)^2}{2}, \\ \frac{2\mu}{1+p}\left(1 - \frac{\xi-s}{1-p}\right) & \text{if } \xi - s \ge \frac{(1-p)^2}{2}. \end{cases}$$

Proof of Claim D.2. I consider the case of $\bar{w} = 2\mu - p$ first, where the distribution in the statement of the claim becomes

$$H_{2\mu-p}(w) = \begin{cases} 0 & w \in [0, p), \\ \frac{w-p}{2(\mu-p)} & w \in [p, 2\mu-p), \\ 1 & w \in [2\mu-p, 1]. \end{cases}$$

Observe that the concave hull of $H_{2\mu-p}\big|_{[0,1-s/\xi]}(p+z)$ coincides with $H_{2\mu-p}(w)$ on $[p,1]$, and hence Nature's value is $H_{2\mu-p}(p+\xi-s)$, which can be obtained by effective outside option distribution with mean $\xi-s$ supported on a subset of $[0,2(\mu-p)]$. Now consider effective outside option distribution

$$\hat{G}^{2\mu-p}(z)=\begin{cases}1-\frac{\xi-s}{\mu-p}+\frac{\xi-s}{2(\mu-p)^2}z & \text{if } z\in[0,2(\mu-p)),\\ 1 & \text{if } z\in[2(\mu-p),1],\end{cases}$$

which induces

$$\hat{G}_p^{2\mu-p}(w)=\begin{cases}0 & \text{if } w\in[0,p)\\ 1-\frac{\xi-s}{\mu-p}+\frac{\xi-s}{2(\mu-p)^2}(w-p) & \text{if } w\in[p,2\mu-p),\\ 1 & \text{if } w\in[2\mu-p,1].\end{cases}$$

Then the concave hull of $\hat{G}_p^{2\mu-p}(w)$ coincide with the function itself on $[p,1]$; and since $\mu>p$, Seller's value is $\hat{G}_p^{2\mu-p}(\mu)=1-(\xi-s)/(2(\mu-p))$. It can be obtained by any distribution with mean μ supported on a subset of $[p,2\mu-p]$, and this condition is satisfied by $H_{2\mu-p}$. Thus, $(H_{2\mu-p},\hat{G}^{2\mu-p})$ is a saddle point; and to show that $H_{2\mu-p}$ is robustly optimal, it only remains to show that $\hat{G}^{2\mu-p}$ can be induced. Consider

$$\nabla(a)=\frac{a}{\mu-p}-1\quad\text{on } [\mu-p,2(\mu-p)];$$

and let $G_a(v)$ be a binary distribution with support on $\{2(\mu-p)-a,\, a+2s(\mu-p)/(\xi-s)\}$. It is not difficult to show that $G_a(v)\in\mathcal{M}(\xi)$ for all $a\in[\mu-p,2(\mu-p)]$; then by Lemma D.1, $\hat{G}^{2\mu-p}$ can be induced.

Next I consider the case of $\bar{w}=\sqrt{(\xi-s)(\xi-s+2p)}+(\xi-s+p)<1$. The concave hull of $H_{\bar{w}}\big|_{[0,1-s/\xi]}(p+z)$ coincides with $H_{\bar{w}}(w)$ on $[p,1]$, and hence Nature's value is $H_{\bar{w}}(p+\xi-s)$, which can be obtained by effective outside option distribution with mean $\xi-s$ and supported on a subset of $[0,\bar{w}-p]$. Now consider effective outside option distribution

$$\hat{G}^{\bar{w}}(z)=\begin{cases}(z+p)/\bar{w} & \text{if } z\in[0,\bar{w}-p),\\ 1 & \text{if } z\in[\bar{w}-p,1],\end{cases}$$

which induces

$$\hat{G}_p^{\bar{w}}(w) = \begin{cases} 0 & \text{if } w \in [0, p) \\ w/\bar{w} & \text{if } w \in [p, \bar{w}), \\ 1 & \text{if } w \in [\bar{w}, 1]; \end{cases}$$

and the concave hull of $\hat{G}_p^{\bar{w}}$ is

$$\widetilde{G}_p^{\bar{w}}(w) = \begin{cases} w/\bar{w} & \text{if } w \in [0, \bar{w}), \\ 1 & \text{if } w \in [\bar{w}, 1]. \end{cases}$$

Consequently, Seller's value is $\widetilde{G}_p^{\bar{w}}(\mu) = \mu/\bar{w}$, which can be attained by any distribution over posteriors with mean μ supported on a subset of $\{0\} \cup [p, \bar{w}]$, and this condition is satisfied by $H_{\bar{w}}$. Thus, $(H_{\bar{w}}, \hat{G}^{\bar{w}})$ is a saddle point; and to show that $H_{\bar{w}}$ is indeed robustly optimal, it only remains to show that $\hat{G}^{\bar{w}}$ can be induced. Consider

$$\nabla(a) = \frac{2a}{\bar{w} - p} - 1 \quad \text{on } [(\bar{w} - p)/2, \bar{w} - p];$$

and let $G_a(v)$ be a ternary distribution with pmf $g_a(v)$ given by

v	0	$\bar{w} - p - a$	$\frac{2\bar{w}\xi}{\bar{w}-p} - (\bar{w} - p) + a$
$g_a(v)$	$p/\bar{w}$	$(1 - p/\bar{w})/2$	$(1 - p/\bar{w})/2$

It is not difficult to show that $G_a(v) \in \mathcal{M}(\xi)$ for all $a \in [(\bar{w} - p)/2, \bar{w} - p]$; then by Lemma D.1, $\hat{G}^{\bar{w}}$ can be induced.

Finally, I consider the case of $\bar{w} = 1$, where the proposed distribution becomes

$$H_1(w) = \begin{cases} 1 - \frac{2\mu}{1+p} & \text{if } w \in [0, p), \\ 1 - \frac{2\mu}{1+p} + \frac{2\mu}{1+p}\left(\frac{w-p}{1-p}\right) & \text{if } w \in [p, 1]. \end{cases}$$

The concave hull of $H_1\big|_{[0,1-s/\xi]}(p + z)$ coincides with $H_1(w)$ on $[p, 1]$, and hence Nature's value is $H_1(p + \xi - s)$, which can be obtained by any effective outside option distribution with mean $\xi - s$ and supported on a subset of $[0, 1 - p]$. Now consider effective outside option distribution

$$\hat{G}^1(z) = \begin{cases} \frac{2(p+z)}{1+p}\left(1 - \frac{\xi - s}{1-p}\right) & \text{if } z \in [0, 1 - p), \\ 1 & \text{if } z \in [1 - p, 1], \end{cases}$$

which induces

$$\hat{G}_p^1(w) = \begin{cases} 0 & \text{if } w \in [0, p) \\ \frac{2w}{1+p}\left(1 - \frac{\xi - s}{1-p}\right) & \text{if } w \in [p, 1]; \end{cases}$$

and the concave hull of $\hat{G}_p^1$ is

$$\widetilde{G}_p^1(w) = \frac{2w}{1+p}\left(1 - \frac{\xi - s}{1-p}\right) \quad \text{on } [0, 1].$$

Consequently, Seller's value is $\widetilde{G}_p^1(\mu)$, which can be attained by any distribution over posteriors with mean μ supported on a subset of $\{0\} \cup [p, 1]$, and this condition is satisfied by H_1. Thus, $(H_1, \hat{G}^1)$ is a saddle point; and to show that H_1 is indeed robustly optimal, it only remains to show that $\hat{G}^1$ can be induced. Consider

$$\nabla(a) = \frac{a - (\xi - s)}{1 - p - (\xi - s)} \quad \text{on } [\xi - s, 1 - p];$$

and let $G_a(v)$ be a quaternary distribution with pmf $g_a(v)$ given by

v	0	$1-p-a$	$\phi - \frac{[1-p-(\xi-s)](1-p-a)}{\xi-s}$	1
$g_a(v)$	$\frac{2p[1-p-(\xi-s)]}{(1-p)(1+p)}$	$\frac{2[1-p-(\xi-s)]^2}{(1-p)(1+p)}$	$\frac{2(\xi-s)[1-p-(\xi-s)]}{(1-p)(1+p)}$	$\frac{2(\xi-s)-(1-p)^2}{(1-p)(1+p)}$

where

$$\phi = \frac{\xi\left(1-p^2\right) + (1-p)^2}{2(\xi - s)} - 1 - \frac{[1-p-(\xi-s)](1-p)}{\xi - s}, \quad \text{and} \quad \tau = \frac{1-p-(\xi-s)}{\xi - s}.$$

It is not difficult to show that $G_a(v) \in \mathcal{M}(\xi)$ for all $a \in [\xi - s, 1 - p]$; then by Lemma D.1, $\hat{G}^1$ can be induced. This completes the proof. ∎

D.3 The case of $p \leq s/\xi$

The following four claims establish the results for this case.

Claim D.3. *If $p \leq s/\xi$ and $\mu \geq p + 1 - s/\xi$, the degenerate distribution δ_μ is optimal. Furthermore, Seller's revenue guarantee when price is p is $\Phi(p) = p$.*

Proof of Claim D.3. By using distribution over posteriors δ_μ, because $\mu - p \geq 1 - s/\xi$, the highest possible effective outside option is below Buyer's net value from purchasing

Seller's product. Therefore, Buyer buys without search with probability one, and thus δ_μ must be optimal, and Seller's revenue guarantee equals her price p. ∎

Claim D.4. *If $p \leq s/\xi$, $\mu < p + 1 - s/\xi$, and $p \geq (1-2\xi)(\xi - s)/(2\xi^2)$, the binary distribution with support on $\{0, p + 1 - s/\xi\}$, is robustly optimal. Its cdf is*

$$H_b(w) = \begin{cases} 1 - \frac{\mu\xi}{\xi(1+p)-s} & \text{if } w \in [0, p + 1 - s/\xi), \\ 1 & \text{if } w \in [p + 1 - s/\xi, 1]. \end{cases}$$

Proof of Claim D.4. Evidently, the concave hull of $H_b\big|_{[0,1-s/\xi]}(p + z)$ is constant on $[p, p + 1 - s/\xi]$, and hence any effective outside option distribution with mean $\xi - s$ supported on a subset of $[0, 1 - s/\xi]$ is optimal for Nature. When $(1-2\xi)(\xi - s)/(2\xi^2) \leq p < (1-2\xi)(\xi - s)/\xi^2$, consider effective outside option distribution

$$\hat{G}^0(z) = \begin{cases} \left(\frac{2}{\xi - s} - \left(\frac{1-\xi}{\xi}\right)\frac{1+\xi}{\xi(1+p)-s}\right) z + \frac{(1-2\xi)(\xi-s)-\xi^2 p}{\xi^2(1+p)-s\xi} & \text{if } z \in [0, \xi - s), \\ \frac{\xi}{\xi(1+p)-s}(z + p) & \text{if } z \in [\xi - s, 1 - s/\xi]; \end{cases}$$

the restriction on p guarantees that $\hat{G}^0$ is a cdf, and it induces

$$\hat{G}^0_p(w) = \begin{cases} 0 & \text{if } w \in [0, p), \\ \left(\frac{2}{\xi - s} - \left(\frac{1-\xi}{\xi}\right)\frac{1+\xi}{\xi(1+p)-s}\right)(w - p) - \frac{(1-2\xi)(\xi-s)-\xi^2 p}{\xi^2(1+p)-s\xi} & \text{if } w \in [p, p + \xi - s), \\ \frac{\xi}{\xi(1+p)-s} w & \text{if } w \in [p + \xi - s, p + 1 - s/\xi), \\ 1 & \text{if } w \in [p + 1 - s/\xi, 1]. \end{cases}$$

Inspection shows that the concave hull of $\hat{G}^0_p(w)$ is

$$\widetilde{G}^0_p(w) = \begin{cases} \frac{\xi}{\xi(1+p)-s} w & \text{if } w \in [0, p + 1 - s/\xi], \\ 1 & \text{if } w \in [p + 1 - s/\xi, 1]. \end{cases}$$

Consequently, any distribution over posteriors with mean μ and with support on a subset of $\{0\} \cup [p + \xi - s, p + 1 - s/\xi]$ is optimal for Seller, and this condition is satisfied by the binary distribution H^b. Thus, $(H^b, \hat{G}^0)$ is a saddle point; to show that H^b is indeed robustly optimal, it only remains to show that $\hat{G}^0$ can be induced. Consider

$$\nabla(a) = \frac{a}{\xi(1 + p) - s} + 1 - \frac{\xi - s}{\xi^2(1 + p) - s\xi} \quad \text{on } [\xi - s, 1 - s/\xi];$$

and let $G_a^0(v)$ be a ternary distribution with pmf $g_a^0(v)$ given by

v	0	$\beta - \gamma a$	$a + s/\xi$
$g_a^0(v)$	$\frac{(1-2\xi)(\xi-s)-\xi^2 p}{\xi^2(1+p)-s\xi}$	p_L	ξ

where

$$p_L = 1 - \xi - \frac{(1-2\xi)(\xi-s)-\xi^2 p}{\xi^2(1+p)-s\xi},$$

$\gamma = \xi/p_L$, and $\beta = (\xi - s)\gamma/\xi$. It can be checked that $G_a^0(v) \in \mathcal{M}(\xi)$ for all $a \in [\xi - s, 1 - s/\xi]$; then by Lemma D.1, $\hat{G}^0$ can be induced.

And for $p \geq (1-2\xi)(\xi-s)/\xi^2$, consider effective outside option distribution

$$\hat{G}^{\xi-s}(z) = \begin{cases} \frac{(1-\xi)^2}{\xi^2(1+p)-s\xi} z & \text{if } z \in [0, \xi - s), \\ \frac{\xi}{\xi(1+p)-s}(z+p) & \text{if } z \in [\xi - s, 1 - s/\xi]; \end{cases}$$

the restriction on p guarantees that $\hat{G}^{\xi-s}$ is a cdf, and it induces

$$\hat{G}_p^{\xi-s}(w) = \begin{cases} 0 & w \in [0, p), \\ \frac{(1-\xi)^2}{\xi^2(1+p)-s\xi}(w-p) & \text{if } w \in [p, p + \xi - s), \\ \frac{\xi}{\xi(1+p)-s} w & \text{if } w \in [p + \xi - s, p + 1 - s/\xi), \\ 1 & \text{if } w \in [p + 1 - s/\xi, 1]. \end{cases}$$

Inspection shows that the concave hull of $\hat{G}_p^{\xi-s}(w)$ is

$$\widetilde{G}_p^{\xi-s}(w) = \begin{cases} \frac{\xi}{\xi(1+p)-s} w & \text{if } w \in [0, p + 1 - s/\xi], \\ 1 & \text{if } w \in [p + 1 - s/\xi, 1]. \end{cases}$$

Consequently, any distribution over posteriors with mean μ and with support on a subset of $\{0\} \cup [p + \xi - s, p + 1 - s/\xi]$ is optimal for Seller, and this condition is satisfied by the binary distribution H^b. Thus, $(H^b, \hat{G}^{\xi-s})$ is a saddle point; to show that H^b is indeed robustly optimal, it only remains to show that $\hat{G}^{\xi-s}$ can be induced. Consider

$$\nabla(a) = \frac{a}{\xi(1+p)-s} + 1 - \frac{\xi - s}{\xi^2(1+p)-s\xi} \quad \text{on } [\xi - s, 1 - s/\xi];$$

and let $G_a^{\xi-s}(v)$ be a binary distribution with pmf $g_a^{\xi-s}(v)$ given by

v	$(\xi(1-a)-s)/(1-\xi)$	$a+s/\xi$
$g_a^{\xi-s}(v)$	$1-\xi$	ξ

It can be checked that $G_a^0(v) \in \mathcal{M}(\xi)$ for all $a \in [\xi - s, 1 - s/\xi]$; then by Lemma D.1, $\hat{G}^{\xi-s}$ can be induced. This completes the proof. ∎

Claim D.5. *If $p \le s/\xi$, $p \le \mu - (1-s/\xi)/2$, and $p < (1-2\xi)(\xi-s)/(2\xi^2)$, the distribution*

$$H^h(w) = \begin{cases} 0 & \text{if } w \in [0, p), \\ 2\frac{\xi^2(p+1-\mu)-s\xi}{(\xi-s)^2}(w-p) & \text{if } w \in [p, p+1-s/\xi), \\ 1 & \text{if } w \in [p+1-s/\xi, 1] \end{cases}$$

is robustly optimal. Furthermore, Seller's revenue guarantee given price p is

$$\Phi(p) = p\left[1 - 2\xi + 2\xi^2\frac{\mu - p}{\xi - s}\right].$$

Proof of Claim D.5. Observe that the concave hull of $H^h\big|_{[0,1-s/\xi]}(p+z)$ coincides with $H^h(w)$ on $[p, p+1-s/\xi]$, which is affine. Consequently, any effective outside option distribution with mean $\xi - s$ supported on a subset of $[0, 1-s/\xi]$ is optimal for Nature. Now consider effective outside option distribution

$$\hat{G}^h(z) = 1 - 2\xi + \frac{2\xi^2}{\xi - s}z \quad \text{on } [0, 1-s/\xi];$$

note that $\hat{G}^h(0) \ge 0$ because $p < (1-2\xi)(\xi-s)/(2\xi^2)$ implies $\xi < 1/2$. Then

$$\hat{G}_p^h(w) = \begin{cases} 0 & \text{if } w \in [0, p), \\ 1 - 2\xi + \frac{2\xi^2}{\xi-s}(w-p) & \text{if } w \in [p, p+1-s/\xi]; \end{cases}$$

and its concave hull is

$$\widetilde{G}_p^h(w) = 1 - 2\xi + \frac{2\xi^2}{\xi - s}(w-p) \quad \text{on } [0, p+1-s/\xi].$$

Therefore, any distribution over posteriors with mean μ and with support on a subset of $[0, p+1-s/\xi]$ is optimal for Seller, and this condition is satisfied by H^h. Furthermore,

Seller's value is $\widetilde{G}_p^h(\mu)$. Thus, $(H^h, \hat{G}^h)$ is a saddle point; and to show that H^h is indeed robustly optimal, it only remains to show that $\hat{G}^h$ can be induced. Consider

$$\nabla(a) = \frac{2\xi}{\xi - s}a - 1 \quad \text{on } [(\xi - s)/(2\xi), 1 - s/\xi];$$

and let $G_a^h(v)$ be a ternary distribution with pmf $g_a^h(v)$ given by

v	0	$1 - s/\xi - a$	$a + s/\xi$
$g_a^h(v)$	$1 - 2\xi$	ξ	ξ

It can be checked that $G_a^h(v) \in \mathcal{M}(\xi)$ for all $a \in [(\xi - s)/(2\xi), 1 - s/\xi]$; then by Lemma D.1, $\hat{G}^h$ can be induced by a mixture of outside option distributions. This completes the proof. ∎

Claim D.6. *If $p \le s/\xi$, $\mu - (1 - s/\xi) \ge p > \mu - (1 - s/\xi)/2$, and $p < (1 - 2\xi)(\xi - s)/(2\xi^2)$, there exists $\bar{w} \in [2\mu - p, 1]$ where*

$$\bar{w} = \begin{cases} 2\mu - p & \text{if } \mu > p \text{ and } \xi - s \le \frac{2(\mu-p)^2}{2\mu-p}, \\ \sqrt{(\xi - s)^2 + 2p(\xi - s)} + (\xi - s + p) & \text{otherwise,} \end{cases}$$

such that the robustly optimal distribution is

$$H_{\bar{w}}(w) = \begin{cases} 1 - \frac{2\mu}{\bar{w}+p} & \text{if } w \in [0, p), \\ 1 - \frac{2\mu}{\bar{w}+p} + \frac{2\mu}{\bar{w}+p}\left(\frac{w-p}{\bar{w}-p}\right) & \text{if } w \in [p, \bar{w}), \\ 1 & \text{if } w \in [\bar{w}, 1]. \end{cases}$$

And Seller's revenue guarantee given price p is

$$\Phi(p) = \begin{cases} p\left[1 - \frac{\xi - s}{2(\mu - p)}\right] & \text{if } \mu > p \text{ and } \xi - s \le \frac{2(\mu-p)^2}{2\mu-p}, \\ \frac{p\mu}{\sqrt{(\xi-s)(\xi-s+2p)}+(\xi-s+p)} & \text{otherwise.} \end{cases}$$

Proof of Claim D.6. In this case,

$$\sqrt{(\xi - s)^2 + 2p(\xi - s)} + (\xi - s + p) < p + 1 - s/\xi \le 1,$$

where the first inequality holds because $p < (1 - 2\xi)(\xi - s)/(2\xi^2)$, and the second follows

from the assumption that $p \leq s/\xi$. Rearranging the above inequalities, $\xi - s < (1-p)^2/2$. The rest of the proof is analogous to the proof of Claim D.2, and hence omitted.[6] ∎

The results in Appendix D.2 and Appendix D.3 together establish Proposition 1.

Appendix E Two Benchmarks: Proofs

E.1 Proof of Proposition 2

When $s = 0$, the definition of a indicates that $a = 1$. Consequently, Seller's payoff when the distribution over outside options is G can be simplified to $p\,\mathbb{E}_G[1 - H(p+v)]$. Therefore, I can work with the outside option distribution directly; in particular, there is no need to find an effective outside option distribution first and then find an outside option distribution that generates it.

For each pair of distribution over posteriors and effective outside option distribution $\left(H, \hat{G}\right)$ that is a saddle point in the proof of Proposition 1, it can be checked that by setting $s = 0$, the resulting pair (H', G') of the distribution over posteriors and outside option distribution is a saddle point for the zero search cost problem. Using these results, an argument similar to the proof of Claim 1 establishes the proposition.

E.2 Proof of Proposition 3

For a fixed posterior w, the probability of Buyer buying from Seller is given by $G_p^a(w)$, where when $p + a \leq 1$,

$$G_p^a(w) := \begin{cases} 0 & \text{if } w < p, \\ G(w-p) & \text{if } p \leq w < p + a, \\ 1 & \text{if } w \geq p + a; \end{cases}$$

and

$$G_p^a(w) := \begin{cases} 0 & \text{if } w < p, \\ G(w-p) & \text{if } w \geq p, \end{cases}$$

[6] There is one subtle point, though: for the case of $\bar{w} = 2\mu - p$, to make sure that my choice of the saddle point in the proof of Claim D.2 works, it has to be that $2(\mu - p) \leq 1 - s/\xi$. This must hold by the assumption that $p > \mu - (1 - s/\xi)/2$.

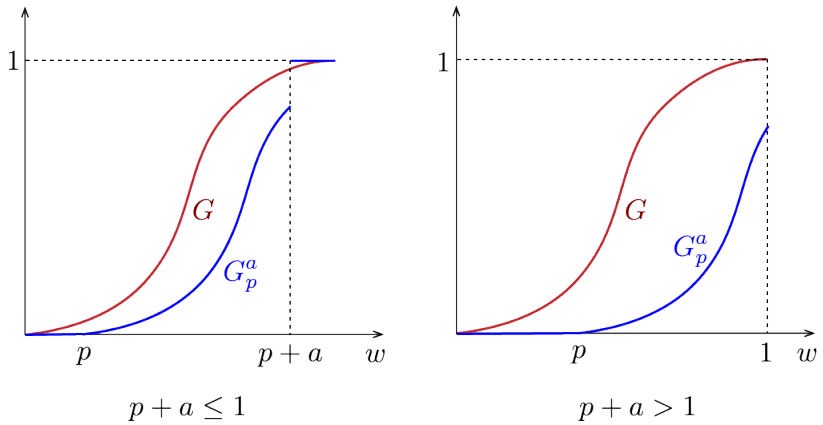

Figure E.1: Constructing G_p^a from G. The left panel shows the case of $p + a \leq 1$, and the case of $p + a > 1$ is displayed in the right panel.

when $p + a > 1$. See Figure E.1 for an illustration of this function. Therefore, if Seller chooses a distribution H, her payoff can be written as $p \int_0^1 G_p^a(w)\,\mathrm{d}H(w)$. Therefore, Seller solves

$$\max_{p\in[0,1]} \left\{ \max_{H\in\mathcal{M}(\mu)} p \int_0^1 G_p^a(w)\,\mathrm{d}H(w) \right\}.$$

To prove Proposition 3, I solve for the optimal information provision policy for a fixed price first. For a fixed $p \in [0, 1]$, Seller's problem of choosing a distribution over posteriors is

$$\max_{H\in\mathcal{M}(\mu)} \int_0^1 G_p^a(w)\,\mathrm{d}H(w).$$

This problem is identical to the information design problem studied in Kamenica and Gentzkow (2011),[7] where Seller's (who plays the role of Sender in their framework) value function is exactly $G_p^a(w)$. Consequently, Seller's optimal distribution can be identified by finding the concave hull of $G_p^a(w)$.

To identify the concave hull of $G_p^a(w)$, starting from $v = 0$, I try to find a line segment that tangents to G_p^a at some $r(p)$: $r(p)$ solves

$$g(r(p) - p)r(p) = G(r(p) - p); \tag{E.1}$$

if a solution to the above equation does not exist for some p, set $r(p) = 1$. Note that when $r(p) < 1$, it must be that $g'(r(p) - p) < 0$. If $p > 1 - a$, it is not hard to see that G_p^a is a convex-concave function on $[0, 1]$, and it is concave on $[r(p), 1]$. Thus, the concave hull

[7] See the problem on page 2596 in Kamenica and Gentzkow (2011).

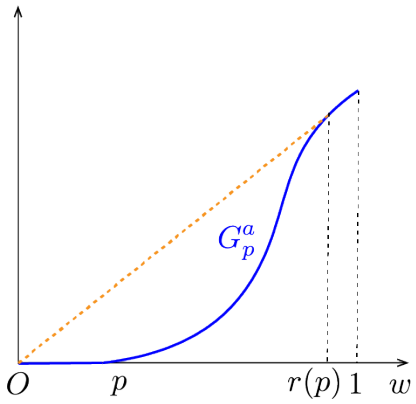

Figure E.2: The concave hull of G_p^a when $p > 1 - a$, which is identified by the orange dashed curve.

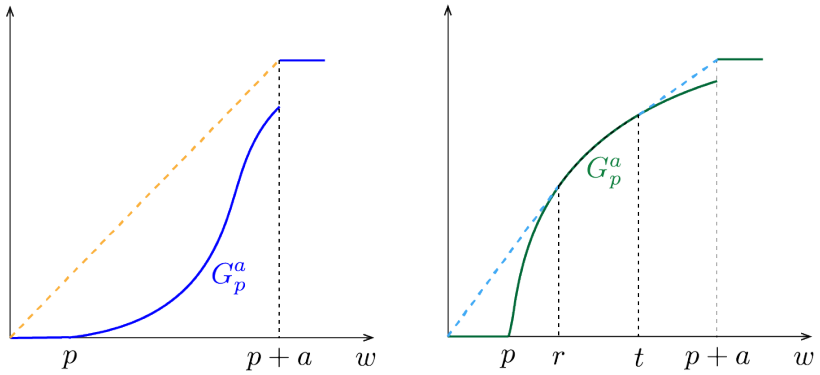

Figure E.3: The concave hull of G_p^a when $p \leq 1 - a$. The left panel depicts the case of $g(r(p) - p) \leq 1/(p + a)$, and the concave hull of G_p^a is identified by the orange dashed curve. The right panel illustrates the case of $g(r(p) - p) > 1/(p + a)$, and the concave hull of G_p^a is identified by the cyan dashed curve.

of G_p^a, as illustrated in Figure E.2, is affine on $[0, r(p)]$, and identical to G_p^a on $[r(p), 1]$.

The case of $p \leq 1 - a$ is more complicated. If $g(r(p) - p) \leq 1/(p + a)$, that is, the slope of G_p^a at $r(p)$ is less than or equal to the slope of the line segment that connects $(0, 0)$ and $(p + a, 1)$, which I denote by ℓ_p, then the concave hull of G_p^a is essentially identified by ℓ_p. If instead $g(r(p) - p) > 1/(p + a)$, then draw another line segment from $(p + a, 1)$ that tangents to G_p^a at some $t(p)$: $t(p)$ solves

$$g(t(p) - p)(p + a - t(p)) = 1 - G(t(p) - p). \tag{E.2}$$

And in this case the concave hull is identified by $r(p)$ and $t(p)$. The left and right panel of Figure E.3 illustrate the above two cases, respectively.

Being able to identify the concave hull of G_p^a, the optimal information provision policy for a fixed price is immediate.

Lemma E.1. *Suppose that $p \leq 1-a$. Then if $g(r(p)-p) \leq 1/(p+a)$, when $\mu \in (0, p+a)$, $\{0, p+a\}$ is optimal;*[8] *and when $\mu \in [p+a, 1]$, $\{\mu\}$ is optimal. If $g(r(p)-p) > 1/(p+a)$,*

- *when $\mu \in (0, r(p))$, $\{0, r(p)\}$ is optimal;*
- *when $\mu \in [r(p), t(p)]$, $\{\mu\}$ is optimal;*
- *when $\mu \in (t(p), p+a)$, $\{t(p), p+a\}$ is optimal; and*
- *when $\mu \in [p+a, 1)$, $\{\mu\}$ is optimal.*

Suppose instead that $p > 1-a$. Then when $\mu \in (0, r(p))$, $\{0, r(p)\}$ is optimal; and when $\mu \in [r(p), 1)$, $\{\mu\}$ is optimal.

Now I am ready to prove Proposition 3.

Proof of Proposition 3. Suppose first that $p > 1-a$. Let

$$\hat{p} = \sup\{p \in [0,1] \,:\, \text{there exists } r(p) \in [0,1] \text{ that solves (E.1)}\}.$$

If G is concave, let $r(0) = 0$; and otherwise let $r(0)$ solve $g(r(0))r(0) = G(r(0))$ if a solution exists, or else let $r(0) = 1$. By the implicit function theorem,

$$r'(p) = r(p) - \frac{g(r(p)-p)}{g'(r(p)-p)}. \tag{E.3}$$

By Lemma E.1, the optimal information provision policy depends on the location of μ: if $\mu \in (0, r(p))$, $\{0, r(p)\}$ is optimal; and if $\mu \in [r(p), 1)$, $\{\mu\}$ is optimal. Suppose $\mu \in (0, r(p))$, Seller's payoff by setting $p \in (1-a, \hat{p})$ is given by

$$p\frac{\mu}{r(p)}G(r(p)-p) = p\mu g(r(p)-p),$$

where the equality follows from (E.1) in the text; and by setting $p \in [\hat{p}, 1)$, Seller's payoff is $p\mu G(1-p)$. It can be checked that, using (E.3), when $p \in (1-a, \hat{p})$ Seller's payoff is

[8] Since every optimal distribution over posteriors is either degenerate or binary, I identify such a distribution by its support.

strictly increasing in p, and hence Seller provides full information by using distribution $\{0, 1\}$, with price being p_h defined by the solution of (25).

Now suppose Seller uses no information $\{\mu\}$; her payoff is given by $pG(\mu - p)$, which is maximized at

$$p_\mu = \frac{G(\mu - p_\mu)}{g(\mu - p_\mu)}. \tag{E.4}$$

Note that by definition of $r(p)$, $r(p) > p$ unless possibly at $p = 0$, which is never optimal. Then

$$\frac{G(r(p_\mu) - p_\mu)}{g(r(p_\mu) - p_\mu)} = r(p_\mu) > p_\mu = \frac{G(\mu - p_\mu)}{g(\mu - p_\mu)},$$

where the first equality follows from (E.1), and the second equality holds by (E.4). By log-concavity of g, it must be that $\mu < r(p_\mu)$. Then the optimal distribution is in fact $\{0, r(p_\mu)\}$, which implies that no information is never optimal. To summarize, when $p > 1 - a$, full information is optimal, and Seller's optimal price and profits are given by p_h and $p_h \mu G(p_h - \mu)$, respectively, where p_h is the solution to (E.2).

Now suppose that $p \leq 1 - a$. Again by the implicit function theorem,

$$t'(p) = 1; \tag{E.5}$$

consequently, both $r(p)$ and $t(p)$ are increasing in p, and by (E.5), $t(p) = t(0) + p$.

If $g(r(0)) \leq 1/a$, it can be checked that for all $p \leq 1 - a$, $g(r(p) - p) \leq 1/(p + a)$. Then by Lemma E.1, the distribution $\{0, p + a\}$ is optimal; and by using this distribution, Buyer buys if and only if she buys without search, which happens with probability $\mu/(p + a)$. Consequently, Seller's expected payoff is $p\mu/(p + a)$, which is strictly increasing in p. It is maximized at $p = 1 - a$, with profit $(1 - a)\mu$; the associated distribution is $\{0, 1\}$, namely full information. To summarize, when $g(r(0)) \leq 1/a$, full information is optimal; $p = 1 - a$ is the optimal price, and Seller's expected payoff is $\mu(1 - a)$.

Consider next the case that $g(r(0)) > 1/a$. Again by Lemma E.1, define

$$\bar{p} = \sup \left\{ p \in [0, 1 - a] \,:\, g(r(p) - p) > \frac{1}{p + a} \right\};$$

by log-concavity of g, $\bar{p}$ is unique. Now for a fixed p, optimal information provision policy again depends on the location of μ: if $\bar{p} \leq p \leq 1 - a$, $\{0, p + a\}$ is optimal; and for $p < \bar{p}$,

- if $\mu \in (0, r(p))$, $\{0, r(p)\}$ is optimal,

- if $\mu \in [r(p), t(p)]$, $\{\mu\}$ is optimal,
- if $\mu \in (t(p), p+a)$, $\{t(p), p+a\}$ is optimal,
- if $\mu \in [p+a, 1)$, $\{\mu\}$ is optimal.

An analogous argument like the case of $p > 1-a$ shows that no information is never optimal; and $p < \bar{p}$ implies that $r(p) < p+a$, but then Seller's expected payoff is strictly increasing in p, which implies that it is strictly better for Seller to price at $\bar{p}$ and provide information according to $\{0, p+a\}$ instead. Consequently, it only remains to consider the third bullet point.

To this end, suppose Seller provides information according to $\{t(p), p+a\}$. Seller's problem of finding the optimal price is

$$p\left[G(t(p)-p)\frac{p+a-\mu}{p+a-t(p)} + \frac{\mu - t(p)}{p+a-t(p)}\right];$$

by (E.2), it can be written as

$$p\left[G(t(0))\frac{p+a-\mu}{a-t(0)} + \frac{\mu - p - t(0)}{a-t(0)}\right];$$

because $G(t(0)) < 1$, it is strictly concave in p. Then the optimal price is given by

$$p_t = \frac{aG(t(0)) - t(0)}{2[1-G(t(0))]} + \frac{\mu}{2}.$$

For this price to be indeed optimal, it has to be that $p_t < \bar{p}$. Observe that at $\bar{p}$, $r(\bar{p}) = t(\bar{p}) = t(0) + \bar{p}$. Then by definition of $\bar{p}$,

$$g(r(\bar{p}) - p) = \frac{G(r(\bar{p}) - \bar{p})}{r(\bar{p})} = \frac{1}{\bar{p}+a},$$

and hence

$$\bar{p} = \frac{aG(t(0)) - t(0)}{1 - G(t(0))}.$$

But then $p_t < \bar{p}$ implies that $\mu < t(0) + p_t = t(p_t)$, which in turn implies that $\{t(p_t), p_t + a\}$ is not optimal at p_t. Therefore, the only candidate for the optimal selling strategy is $(p, \{0, p+a\})$. Consequently, similar to the case of $g(r(0)) \leq 1/a$, full information is optimal, $p = 1-a$ is the optimal price, and Seller's expected payoff is $\mu(1-a)$. Summarizing, when

$p \leq 1 - a$, the selling strategy described above is optimal.

To conclude, full information is always optimal, the choice of the optimal selling strategy boils down to comparing $1 - a$ and $p_h G(1 - p_h)$. Then $p = 1 - a$ is optimal if and only if $p_h G(1 - p_h) \leq 1 - a$, and otherwise p_h is optimal. This yields the statement in the proposition, and hence concludes the proof. ∎

E.3 Proof of Corollary 1

Because the density of the outside option distribution g is strictly positive, the definition of a indicates that it is strictly decreasing in s. Because p_h does not depend on s, there exists a unique a^* that solves $1 - a = p_h G(1 - p_h)$; let the search cost that corresponds to a^* be $\hat{s}_G$. Then the statement follows from Proposition 2.

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