Costly Evidence and Discretionary Disclosure
Supplementary Appendix (For Online Publication)

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Contents

The purpose of this appendix is to establish that the main results of Whitmeyer and Zhang (2022) persist in the alternative setting of Dye (1985) and Jung and Kwon (1988) as well as when we impose a more general cost of information acquisition. That is, in either extension, our three main findings endure:

1. When evidence acquisition is costly, transparency in the acquisition process is negative as it grants the sender a form of commitment to acquire less information.

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2. The receiver always weakly (and sometimes strictly) prefers a positive experiment failure rate, in stark contrast to models with exogenous evidence.
3. Cheap information is different from free information: allowing evidence acquisition costs to vanish selects for the Pareto-worst free-information equilibrium.

A Random Experiment Failure

Now, there is no certification cost but with probability ρ the sender's experiment fails and returns a null finding (her evidence endowment is $\{m_\varnothing\}$).

A.1 Costless Evidence Benchmark

First, we set $\kappa = 0$. This is a special case of Shishkin (2022). Given a conjectured (expected) value of non-disclosed evidence, α, the sender's payoff from acquiring posterior x is

$$V(x) = \begin{cases} \alpha, & \text{if} \quad 0 \leq x < \alpha \\ (1-\rho)x + \rho\alpha, & \text{if} \quad \alpha \leq x \leq 1 \end{cases},$$

where $\alpha \leq \mu$. Like when there is a certification cost, this function is convex and piecewise linear, with one inflection point at α. Again, the sender will not pool any posteriors below α with those above when she acquires information. Thus, α solves

$$\alpha = \frac{\int_0^\alpha x dF(x) + \rho \int_\alpha^1 x dF(x)}{F(\alpha)(1-\rho) + \rho}, \tag{•}$$

or

$$\alpha\left[F(\alpha)(1-\rho) + \rho\right] - \int_0^\alpha x dF(x) - \rho \int_\alpha^1 x dF(x) = 0.$$

The left-hand side of this equation is strictly increasing in α, is negative when $\alpha = 0$, and is strictly positive when $\alpha = \mu$. Thus, a fixed-point exists and is unique.

Moreover, like the costly certification case, there is a multiplicity of equilibria: the sender is indifferent about any local MPC on $[0, \alpha]$ or $[\alpha, 1]$ and so an exact analog of Proposition 3.2 holds:

Proposition A.1. *With covert evidence acquisition (and $\kappa = 0$), any evidence acquisition protocol in which the sender does not pool states above α (defined in •) with those below, then discloses a posterior mean, x, if and only if $x \geq \alpha$, is an equilibrium. There are no other equilibria.*

These equilibria can be Pareto ranked: the Pareto-best equilibrium is that in which the sender acquires full information and discloses above α (whenever possible). The Pareto-worst equilibrium is that in which the distribution of posterior means above α is degenerate on $\mathbb{E}[\theta | \theta \geq \alpha]$.

On the other hand, it is clear that with overt evidence acquisition, the situation with exogenous failure is different from that with costly certification. Indeed, it is clear that for any covert equilibrium there is an outcome-equivalent overt equilibrium, i.e., there exists an overt equilibrium in which the sender acquires the same distribution over posterior means and the same disclosure rule as in the corresponding covert equilibrium.

Proposition A.2. *When $\kappa = 0$, any covert equilibrium is also an equilibrium with overt evidence acquisition.*

Proof. The proof is immediate: the sender is getting her maximal payoff (μ) on path. It is clear that she cannot benefit by deviating to a different evidence acquisition protocol, provided the receiver revises her beliefs (which she can do) about the disclosure protocol following the first-stage deviation. ■

On the other hand, it is easy for us to characterize the Pareto-worst equilibrium, which is worse for the receiver than any of the overt equilibrium.

Lemma A.3. *When evidence acquisition is overt, there exist equilibria in which the sender acquires no evidence. These are the Pareto-worst equilibria.*

Proof. The result is trivial. The sender gets her maximal payoff and the receiver her minimal one. ■

A.2 The Persistence of Points 1 and 3

Now let us specify that $\kappa > 0$. By a virtually identical argument to that for Lemma 5.1, for all sufficiently small $\kappa > 0$, all covert acquisition equilibria are monotone partitional.

If the equilibrium is monotone partitional and involves information acquisition, the sender solves

$$\max_{t} \left\{ F(t)(\alpha - \kappa c(x_L(t))) + (1-\rho) \int_t^1 a dF(a) + (1 - F(t))(\rho\alpha - \kappa c(x_H(t))) \right\},$$

where $x_L(t) := \mathbb{E}[\theta | \theta \le t]$ and $x_H(t) := \mathbb{E}[\theta | \theta > t]$. Taking the FOC then setting $\kappa = 0$ and replacing α in the FOC with x_L, we obtain $x_L = t$, which is precisely the Pareto-worst costless evidence acquisition equilibrium. Thus, we have obtained an analog to Theorem 5.2:

Theorem A.4. *When evidence acquisition is covert and subject to exogenous failure, taking any sequence of equilibria as $\kappa \downarrow 0$ selects the Pareto-worst equilibrium when evidence is free.*

A similarly pernicious result holds when evidence acquisition is overt.

Proposition A.5. *When evidence acquisition is overt and subject to exogenous failure, taking any sequence of equilibria as $\kappa \downarrow 0$ yields the Pareto-worst equilibrium when evidence is free.*

Proof. In fact, a stronger result holds. The only equilibria that exist when $\kappa > 0$ and evidence acquisition is overt are those in which the sender acquires no information. The result is obvious: it is only by acquiring nothing that the sender obtains her maximal payoff of μ. ■

Thus, we see that cheap information is different (and worse) than free information (point 3) and that when $\kappa > 0$ overt evidence acquisition is worse for the receiver than covert (point 1).

A.3 The Persistence of Point 2

It suffices to establish an analog of Proposition 4.7.

Proposition A.6. *If the failure rate, ρ, is 0, but evidence gathering is costly, $\kappa > 0$, the unique equilibrium is that in which the sender acquires no evidence both when evidence acquisition is covert and when it is overt.*

Proof. The overt acquisition proof is immediate. When evidence acquisition is covert, observe that there can be no equilibria in which the sender does not disclose with positive probability since acquiring a posterior mean equal to α (the conjectured non-disclosure value) is never optimal. If the sender discloses with probability 1 she must not acquire any information since her value function is strictly concave on $[0, 1]$. This is obviously an equilibrium. ■

As in Proposition 4.7, the value function for the sender must be locally strictly convex at the conjectured posterior α.

B A More General Cost of Acquiring Information

Now, we reintroduce the certification cost but allow for a more general cost of acquiring information. Specifically, now we stipulate that the sender's cost of acquiring information can be written as the expectation of a convex function of the first z (non-centered) moments

$$m_1 = \int_0^1 x dH(x), \quad m_2 = \int_0^1 x^2 dH(x) \quad \dots \quad m_z = \int_0^1 x^z dH(x) \ ,$$

where H is the posterior distribution upon observing some signal realization.

Associated with the prior F is a joint distribution over $\left(x, x^2, \dots, x^z\right)$, Φ, and the sender's problem of acquiring a signal about the state is equivalent to choosing a fusion (the n-dimensional version of an MPC), Ψ, of Φ. The cost of acquiring such a distribution is

$$C(\Psi) = \kappa \int_{[0,1]^z} c(m_1, \dots, m_z) \, d\Psi(m_1, \dots, m_z),$$

where $\kappa > 0$ is a scaling parameter and c is strictly convex function that satisfies $c(\mu_1, \dots, \mu_z) = 0$ and is bounded on $(0,1)^z$.[1] Note that the payoff (gross of the cost of acquiring information) is simply the posterior first moment as in the main specification.

Consequently, §3 (the $\kappa = 0$ benchmark) in the text is also the benchmark here.

B.1 The Persistence of Points 1 and 3

From Kleiner et al. (2022), the sender's optimal information acquisition corresponds to a power diagram (a particular form of tiling). Moreover, it is either convex partitional, in which $(0,1)^z$ is partitioned into two convex sets and Φ is collapsed on each to its respective barycenters, or not.

By an analogous argument to that for Lemma 5.1, one can show that for all sufficiently small $\kappa > 0$, all covert acquisition are not only convex partitional, but are induced uniquely by a truncation of the prior F at some interior point $t \in (0,1)$. Indeed, the optimal fusion will have binary support, and mirroring the proof of Lemma 5.1, one can show that unless one truncates the distribution, the support points cannot be sufficiently extreme.

If the equilibrium is of this truncation form, the sender solves

$$\max_t \left\{ F(t)\left(\alpha - \kappa \hat{c}(x_L(t))\right) + \int_t^1 a dF(a) - (1 - F(t))\left(\gamma + \kappa \hat{c}(x_H(t))\right) \right\},$$

[1]This type of multi-moment information design/acquisition problem is studied in Kleiner et al. (2022). They also introduce the technique of reinterpreting the problem as a multi-dimensional problem.

where $x_L(t) := \mathbb{E}[\theta | \theta \leq t]$ and $x_H(t) := \mathbb{E}[\theta | \theta > t]$; and where $\hat{c}(\cdot)$ is the cost function corresponding to $c(\cdot, \dots, \cdot)$, which is clearly well-defined (since the sender is truncating F).

Taking the FOC then setting $\kappa = 0$ and replacing α in the FOC with x_L, we obtain $x_L = t - \gamma$, and so the limiting truncation point, t, is exactly that in the costless evidence benchmark. Moreover, for arbitrary small $\kappa > 0$, the sender's posterior distribution is binary. Thus, the limit distribution is the Pareto-worst distribution described in Proposition 3.2.

Proposition B.1. *Take any sequence of equilibria as $\kappa \downarrow 0$. The limit equilibrium is the Pareto-Worst equilibrium when evidence is free.*

It is easy to see; moreover, that the superiority of covert to overt information acquisition also continues (point 1).

B.2 The Persistence of Point 2

An exact match to Proposition 4.7 holds:

Proposition B.2. *If certification is costless, $\gamma = 0$, but evidence gathering is costly, $\kappa > 0$, the unique equilibrium is that in which the sender acquires no evidence but gets it certified.*

Proof. The proof is identical: at the conjectured no-certification value $\alpha > 0$, the sender's payoff is (locally) strictly convex and hence it cannot be a point of support in an equilibrium. ■

Figure 1 illustrates this local convexity at $x = \alpha$ when the cost functional is

$$c(x, y) = \left(x^2 - y\right)^2 - \left(\frac{1}{4} - \frac{1}{3}\right)^2.$$

References

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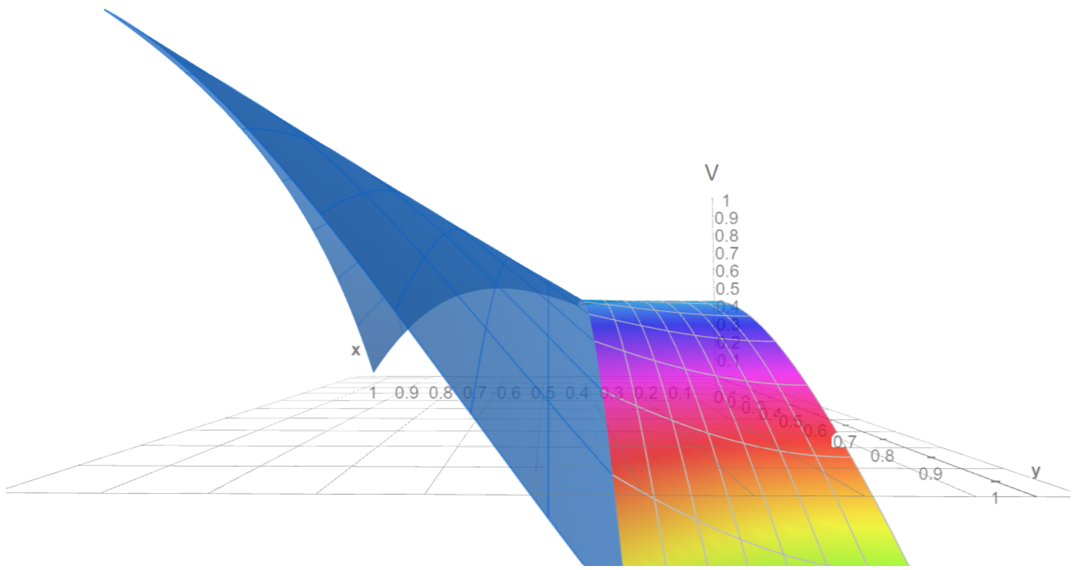

Figure 1: The local convexity of $V(x, y)$ ($x := m_1$, $y := m_2$) at $x = \alpha$.

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